Double Integration Method

نسألكم الدعاء

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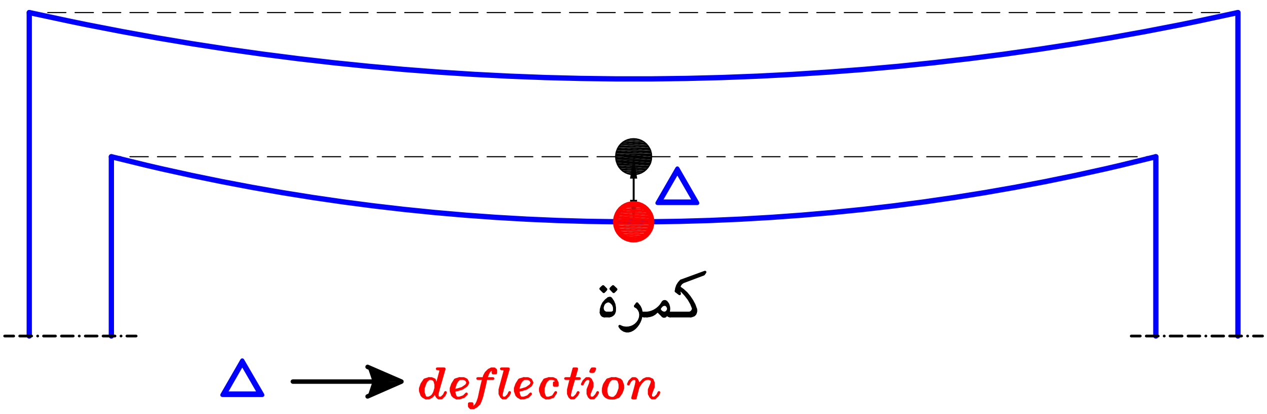
DEFLECTION OF STRUCTURES

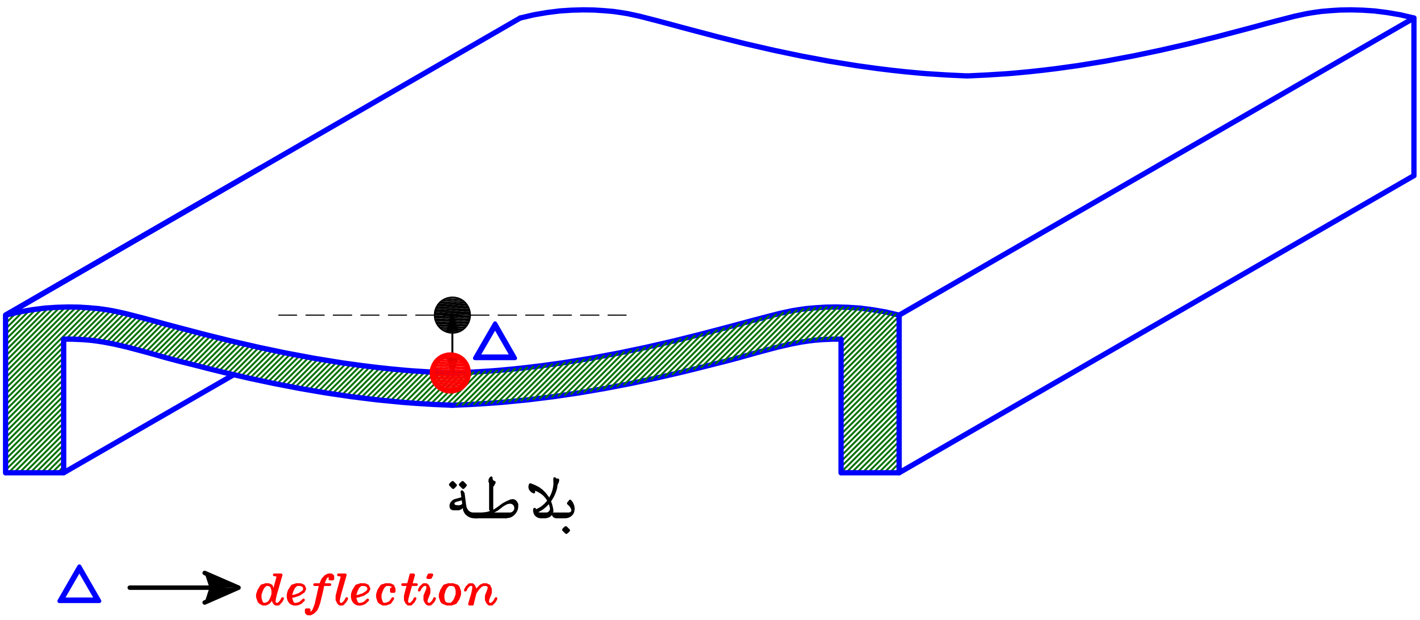
أى منشأ يحدث له حركة طفيفة نتيجة التأثير عليه بأى *Load* أو نتيجة تغير فى درجات الحرارة أو الـ *Shrinkage* .

الـ*deflection*(الترخيم) هو عبارة عن حدوث حركة للنقطة من مكانها الاصلى الى مكان جديد .

مقاومة أى منشأ لحدوث الحركة يسمى *Stiffness* وهذه سوف يتم دراستها فى سنة (٣مدنى) .

الـ *deflection* اما أن يكون *Linear deflection* أو *Rotational deflection* .





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أهمية معرفة و حساب الـ *deflection*

١- حدوث *deflection* زائد فى الكمرات مثلا يؤدى الى حدوث شروخ بالكمرة و بالتالى العمل على صدأ حديد التسليح أ و تشقق البياض نتيجة للشروخ .

٢- حدوث *deflection* زائد لكمرات و عمدان أى مبنى من الممكن أن يؤثر على الحوائط الداخلية للمبنى و يعمل على تكسيرها .

٣- يستخدم الـ *deflection* فى ايجاد معادلات نستخدمها لحل المنشأ اذا كانت معادلات الاتزان غير كافية للحل و ها سوف ندرسه فى الترم الثانى .

For statically determinate structures:

بالنسبة لهذه المنشآت يكون *NO. of unknowns=NO. of equations*

الـ*Unknowns* هى عبارة عن الـ*Reactions* و الـ*Forces* فى الـ*Link members* .

الـ*Equations* هى عبارة عن الـ*Equilibrium equations* و الـ *I.H. Equations* .

$(\sum X = 0, \sum Y = 0, \sum M \text{ @ any point } = 0) + \text{I.H. Equations}$

For statically indeterminate structures:

بالنسبة لهذه المنشآت يكون *NO. of unknowns > NO. of equations* و بالتالى لحل هذه المنشآت نحتاج الى معادلات زيادة لكى تصبح عدد المعادلات مساوى لعدد المجاهيل و هذه المعادلات ممكن أن نحصل عليها عن طريق معرفة قيم الـ*deflection* و الـ*Slope angle* عند بعض النقط و هذا ما سوف نتعلمه الترم القادم أما هذا الترم سنتعلم كيفية حساب الـ*deflection* و الـ*Slope angle* .

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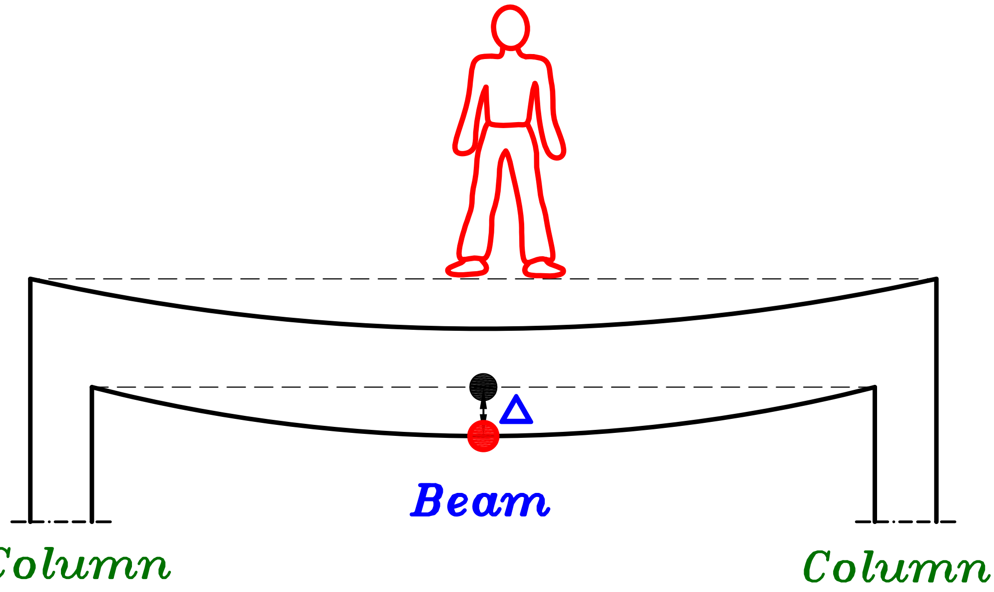
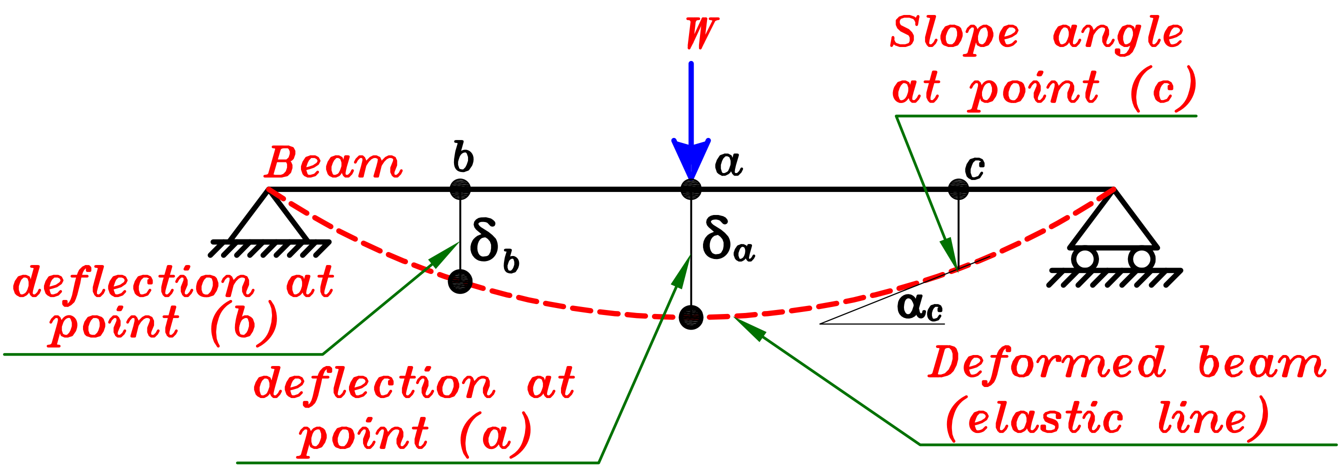
DEFLECTION OF STRUCTURES





ال *deflection* هو تحرك النقطة من مكانها الاصلى الى مكان جديد نتيجة التأثير عليها بأحمال و يرمز له اما بـ Y أو δ .

δ_b & δ_a ⟶ *deflectioms at points (a & b)*

ال *Slope angle* هو دوران النقطة من وضعها الاصلى الى وضع جديد أو هو ميل النقطة فى وضعها الجديد عن وضعها الاصلى و يرمز له بـ α أو Y' .

α_c ⟶ *slope angle at the point (c).*

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Methods of calculating deflections :

1 – *The double Integration method.*
   (Maculay's method)

2 – *The conjugate beam method.*
   (The moment area method)

3 – *The elastic–load method.*

4 – *The method of virtual work.*

5 – *Graphical method for truss deflection.*

الطرق التى سيتم دراستها

بعض هذه الطرق يستخدم لحساب الـ *deflection* عند نقطة معينة و البعض الاخر يحسب الـ *deflection* عند أى نقطة على الـ *Member* .

بعض هذه الطرق يأخذ فى اعتباره تأثير الـ *Bending* و البعض الاخر يأخذ تأثير الـ *Bending & Normal & Shear* .

حساب الـ *deflection* له أهمية كبيرة فى مجالات التصميم حيث أن الاكواد الخاصة بالتصميم تنص على عدم تخطى قيم الـ *deflection* لقيم معينة تنص عليها و ذلك للاغراض الانشائية و البصرية أو النفسية .

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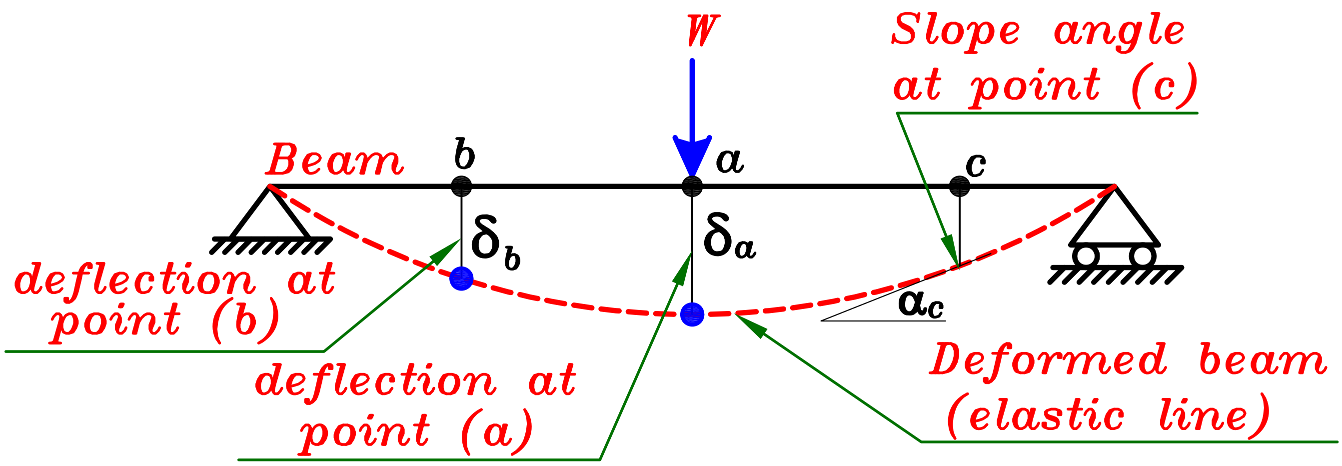
DOUBLE INTEGRATION METHOD

تعتمد هذه الطريقة على حل معادلة تفاضلية من الدرجة الثانية بين العزوم و التفاضل الثانى للـ *deflection* و عن طريق عمل تكامل مرتين لهذه المعادلة نحصل على الـ *deflection* .



كل فكرة هذه الطريقة هى كتابة معادلة الـ *Moment* بطريقة معينة سيتم توضيحها
ثم بتكامل هذه المعادلة نحصل على الـ *Slope angle (Y`)*
وبتكاملها مرة أخرى نحصل على *Deflection (Y)* .

$$\boxed{\frac{E}{R} = \frac{M}{I} = \frac{f}{y}}$$

E = young's modulus of elasticity.
I = moment of inertia of the beam cross section.
M = bending moment equation.
R = Radius of curvature
f = normal stress
y = ordinate of fibre at which (f) is determined.
بعد النقطة التى نحسب عندها الـ *Stress* .

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$$\frac{E}{R} = \frac{M}{I}$$

$M = E\ I\ /\ R$ --- if $1\ /\ R = K =$ curvature $= y''$

$M = E\ I\ K = E\ I\ y''$

$y'' = (\ 1\ /\ EI\ )\ M$

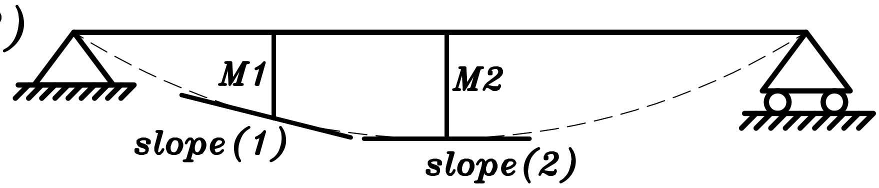
$y'' = d^2y\ /\ dx^2$ & $y = dy\ /\ dx$

$$\boxed{\text{Curvature } (\ y''\ ) = d^2y/dx^2 = -\ (M/EI)}$$

الاشارة السالبة لانه كلما زادت X يقل ال Slope (dy/dx)

$M1 > M2$

Slope (1) < Slope (2)



by integrating this equation we can get the slope angle

$$\boxed{\text{SLOPE } (y') = dy/dx = \frac{1}{EI}\left[\int(-M\ dx)+C_1\right]}$$

by integrating this equation we can get the deflection

$$\boxed{\text{DEFLECTION} = y = \frac{1}{EI}\left\{\int\left(\int(-M\ dx)+C_1\right)+C_2\right\}}$$

Where:

E = young's modulus of elasticity.

I = moment of inertia of the beam cross section.

M = bending moment equation.

C_1&C_2 can be found from boundary(intial) conditions.

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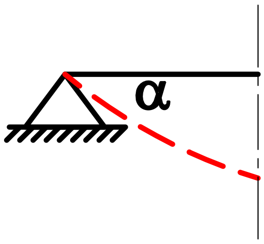
SUPPORTS AND BOUNDARY CONDITIONS

ال boundary conditions هى معلومات تكون معروفة عند نقط معينة ولكل Support معلومات نعرفها عنه و هى كالتالى :

1) Hinged Support



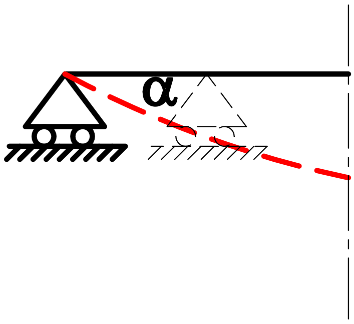
يسمح بالدوران $\alpha \neq 0$

يمنع الحركة فى اتجاه X $x = 0$

يمنع الحركة فى اتجاه Y $y = 0$

حيث أن (α) هى الـ Slope angle و ال (x) هى حركة النقطة فى اتجاه محور (X) اى الاتجاه الافقى و ال (y) هى الحركة فى اتجاه محور (Y) اى الاتجاه الرأسى .

2) Roller Support

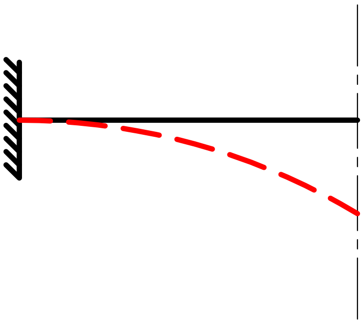


يسمح بالدوران $\alpha \neq 0$

لا يمنع الحركة فى اتجاه X $x \neq 0$

يمنع الحركة فى اتجاه Y $y = 0$

3) Fixed Support



لا يسمح بالدوران $\alpha = 0$

يمنع الحركة فى اتجاه X $x = 0$

يمنع الحركة فى اتجاه Y $y = 0$

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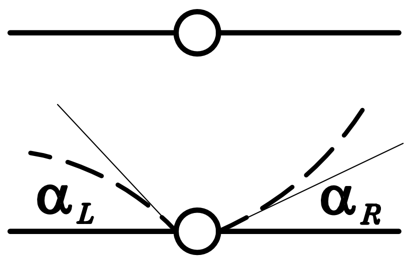
4) Intermediate hinge



$\alpha_L \neq \alpha_R \neq 0$ يسمح بالدوران

$x \neq 0$ X لايمنع الحركة فى اتجاه

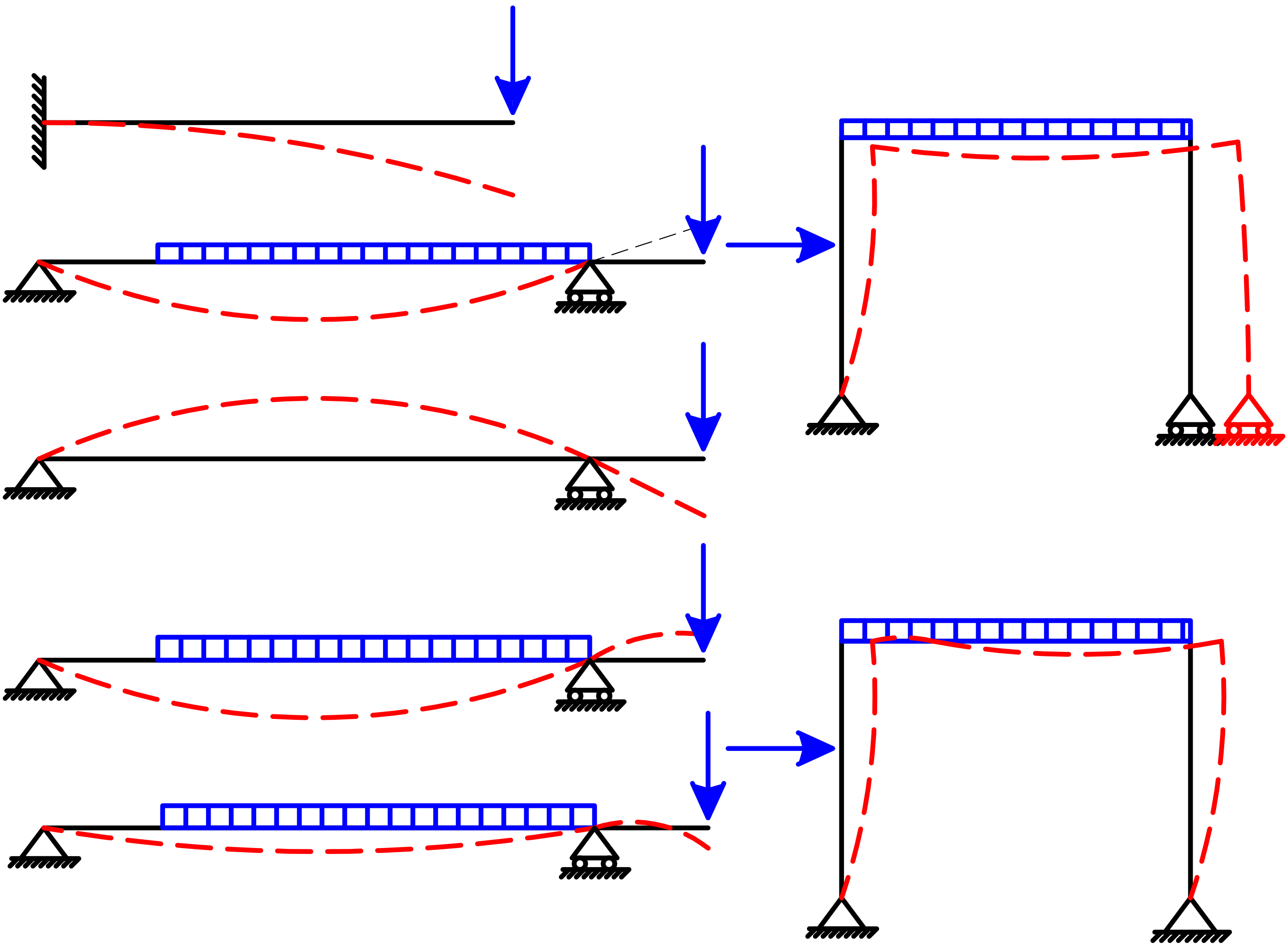
$y \neq 0$ Y لايمنع الحركة فى اتجاه

ELASTIC LINE

هو شكل الكمرة بعد حدوث الـ *Deformations* لها .

ملحوظة

تحتفظ الـ*Rigid Joint* بزواياها بعد الـ*Deformation*



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١ - أن تكون ال *Inertia (I)* ثابتة خلال طول التكامل .

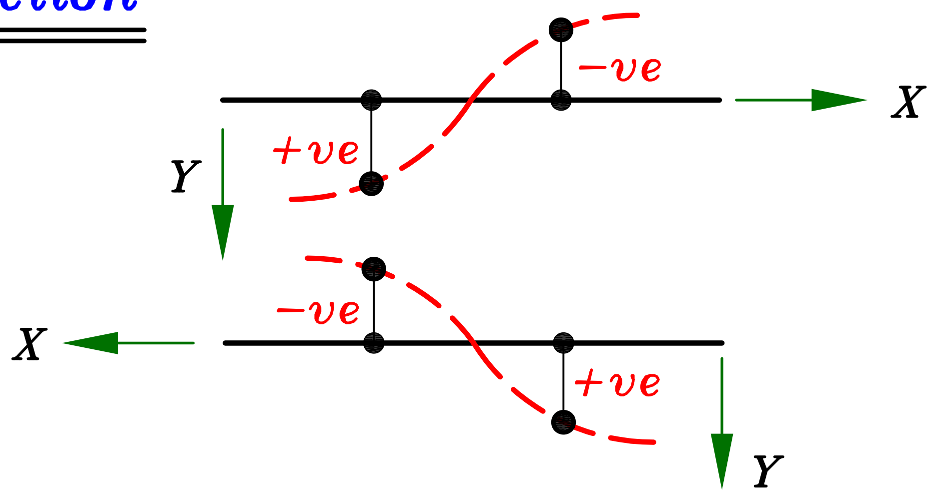
٢ - المعادلة التفاضلية و تكاملاتها تكون متصلة خلال طول التكامل .
أى أنه لا يوجد مثلا *Intermediate hinge* .

ملحوظة

فى حالة وجود *(I.H.)* أو تغير فى *(I)* نقوم بتجزئة المسألة الى أكثر من جزء عند أماكن تغير ال *(I)* أو ال *(I.H.)*

قاعدة الاشارات :

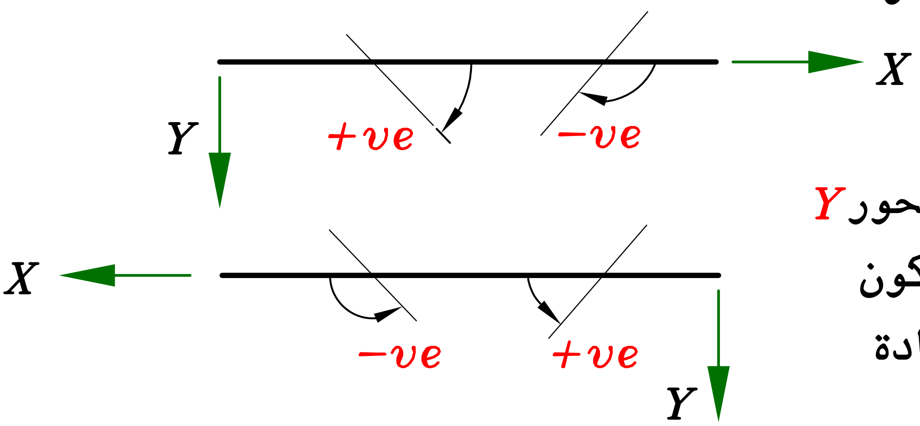
1) Deflection



لو ال *deflection* لاسفل يأخذ اشارة $+ve$ ولو لاعلى يأخذ اشارة $-ve$

2) Slope angle (Rotation) (α)

نتحرك من الوضع الاصلى للكمرة
الى المماس للنقطة التى
نحسب عندها ال α
فى الاتجاه من محور X الى محور Y
فاذا كانت الزاوية منفرجة تكون
الاشارة $+ve$ و اذا كانت حادة
تكون $-ve$



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كيفية الحصول على معادلة الـMoment

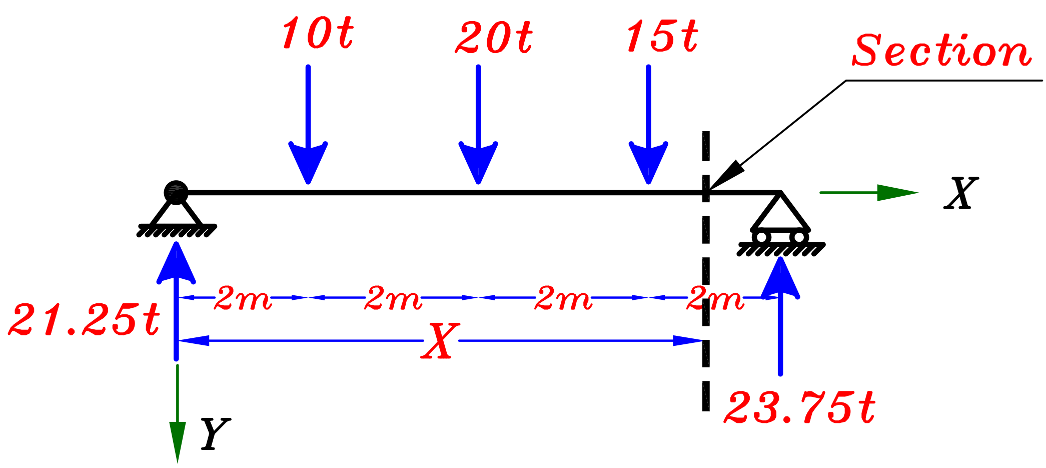
لابد من اختيار (Section) يمثل الكمرة كلها و لذلك يجب ان يكون الـ (Section) قبل اخر الكمرة مباشرة و يبعد عن بدايتها مسافة (X) و تكون المعادلة دالة فى (X) .

و المقصود بان الـ Section يعبر عن الكمرة كلها ان كتابة معادلة الـ moment كمعادلة دالة فى (X) و هى المسافة من بداية الكمرة تصلح للتطبيق عند أى نقطة فى الكمرة و حيث أننا تعلمنا فى العام السابق أنه يمكن كتابة معادلة الـ moment من يمين الـ Section أو يساره لذلك نختار الـ Section فى نهاية الكمرة حتى يعبر عن الكمرة كلها .

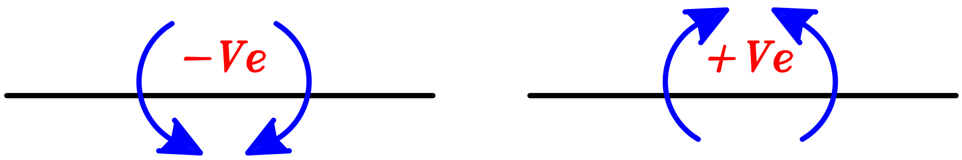
١- يتم وضع المحاور (محور (Y) لاسفل و محور (X) لليمين أو لليسار) ثم يتم تحديد مكان الـ (Section) .

ملحوظة هامة فى المحاضرة يتم أخذ محور (X) لليمين دائما .

الـ Section قبل نهاية الكمرة مباشرة و المقصود ببداية الكمرة هو تقاطع محورى X&Y .



٢- يتم كتابة معادلة العزوم فى (X) مع الاخذ فى الاعتبار اشارات العزوم .



معادلة العزوم

$$\boxed{M(x)=21.25(x)-10(x-2)-20(x-4)-15(x-6)}$$

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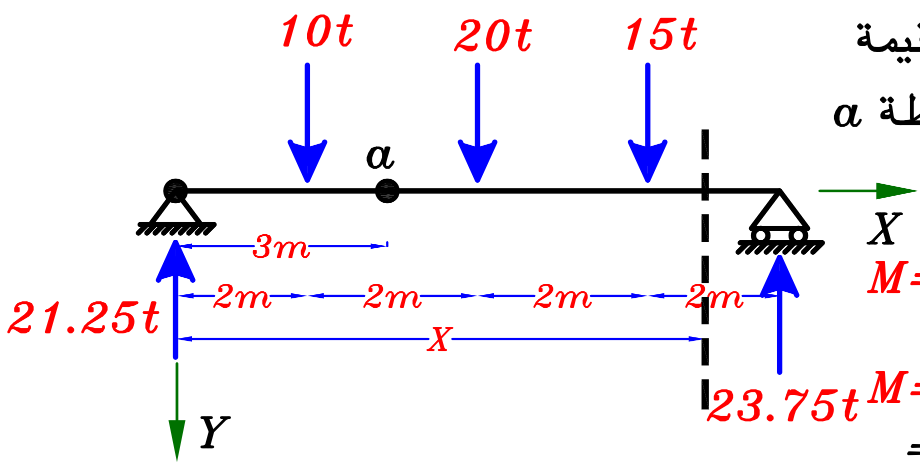
اذا تم حساب العزوم عند قطاع على بعد ٣متر من بداية الكمرة نجد ان ال (21.25t&10t) فقط هى التى تدخل فى المعادلة و ان باقى ال *Loads* لا تدخل فى و ذلك لان باقى الـ*Loads* تقع بعد القطاع و لكن المعادلة لا تفهم و لتفادى مثل هذه المشكلة يجب عدم فك الاقواس فى معادلة العزوم و عند التعويض فى المعادلة بأى مسافة اذا وجد ما بداخل الاقواس(-ve)يتم اعتبار القوس مساويا للصفر و اذا كانت الاشارة (+ve) هذا يعنى أننا نأخذ القوس معنا .

و الفكرة كلها أننا نريد أن تكون المعادلة صالحة لحساب ال *moment* عند أى نقطة و تعطى قيمة ال *moment* الحقيقية .

فمثلا فى هذه الكمرة لحساب قيمة ال *moment* الصحيحة عند نقطة a نحسب من اليمين أو اليسار



$M=21.25x3-10x1=53.75m.t$
من اليسار

$M=23.75x5-15x3-20x1$
$=53.75m.t$ من اليمين

و الان نجرب المعادلة و نشوف حتدينا نفس القيمة أم لا حيث نعوض عن $x=3$

$$M(x)=21.25(x)-10(x-2)-20(x-4)-15(x-6)$$

$$M(x)=21.25(3)-10(3-2)-20(3-4)-15(3-6) = 118.75m.t$$

وهذه الاجابة خاطئة و لكن مع تطبيق قاعدة عدم فك الاقواس و أن مابداخل القوس لو $-Ve$ أو صفر نحذف هذا الترم

$$M(x)=21.25(3)-10(3-2)-\cancel{20(3-4)}-\cancel{15(3-6)}$$

$$M(x)=21.25(3)-10(3-2) = 53.75m.t$$

و بهذا تكون الاجابة صحيحة و تعتبر هذه أهم قاعدة فى هذا الدرس

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٣- يتم كتابة معادلة العزوم و تفاضلها الاول و الثانى .

$$EIy'' = -M$$

$$EIy'' = 15\ (x-6) + 20\ (x-4) + 10\ (x-2) - 21.25\ (x)$$

$$EIy' = (15/2)\ (x-6)^2 + (20/2)\ (x-4)^2 + (10/2)\ (x-2)^2 - (21.25/2)\ (x)^2 + C_1$$

$$EIy = (15/6)\ (x-6)^3 + (10/3)\ (x-4)^3 + (5/3)\ (x-2)^3 - (21.25/6)\ (x)^3 + C_1 x + C_2$$

From boundary conditions

$at\ X = 0\ m \Longrightarrow Y=0$

$at\ X = 8\ m \Longrightarrow Y=0$

AT X = 0

$$EIy = 0 = (15/6)\ \overset{-ve}{\cancel{(0-6)}}^3 + (10/3)\ \overset{-ve}{\cancel{(0-4)}}^3 + (5/3)\ \overset{-ve}{\cancel{(0-2)}}^3 - (21.25/6)\ (0) + 0 + C_2$$

$$\boxed{C_2 = 0}$$

AT X = 8

$$EIy = 0 = (15/6)\ (8-6)^3 + (10/3)\ (8-4)^3 + (5/3)\ (8-2)^3 - (21.25/6)\ (8) + 8\ C_1 + 0$$

$$0 = 20 + 213.33 + 360 - 1813.33 + 8\ C_1$$

$$\boxed{C_1 = 152.50}$$

و بالتعويض بقيمة الثوابت C_1 & C_2 نحصل على معادلتى ال *deflection*
و ال *Slope angle* بدلالة ال *(X)*

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$$EIy' = (15/2)\ (x-6)^2 + (20/2)\ (x-4)^2 + (10/2)\ (x-2)^2 - (21.25/2)\ (x)^2 + 152.50 \longrightarrow (1)$$

$$EIy = (15/6)\ (x-6)^3 + (10/3)\ (x-4)^3 + (5/3)\ (x-2)^3 - (21.25/6)\ (x) + 152.50\ x \longrightarrow (2)$$

للحصول على قيمة الـ *deflection* أو الـ *Slope angle* عند أى نقطة نعوض فى المعادلتين السابقتين بقيمة الـ *(X)* لهذه النقطة فالمعادلة الاولى تعطى الـ *Slope angle* و الثانية تعطى الـ *deflection* .

ملحوظة هامة جدا جدا

فى أى من المعادلات السابقة اذا وجد ما بداخل الاقواس *(-ve)* يتم اعتبار القوس مساويا للصفر و اذا كانت الاشارة *(+ve)* هذا يعنى أننا نأخذ القوس معنا فى الحسابات .

و لكن توجد الكثير من الحالات الخاصة لمعادلة الـ *Moment* و التى للتغلب عليها سوف نضطر الى التلاعب على معادلة الـ *Moment* حتى نتمكن من استخدامها .

وهذا التلاعب حتى تحقق معادلة الـ *Moment* الكمرة كلها و لذلك للتأكد فى أى مرة من صحة المعادلة نعوض فيها عند أكثر من نقطة و نتأكد أن الناتج يكون هو قيمة الـ *Moment* الحقيقية .

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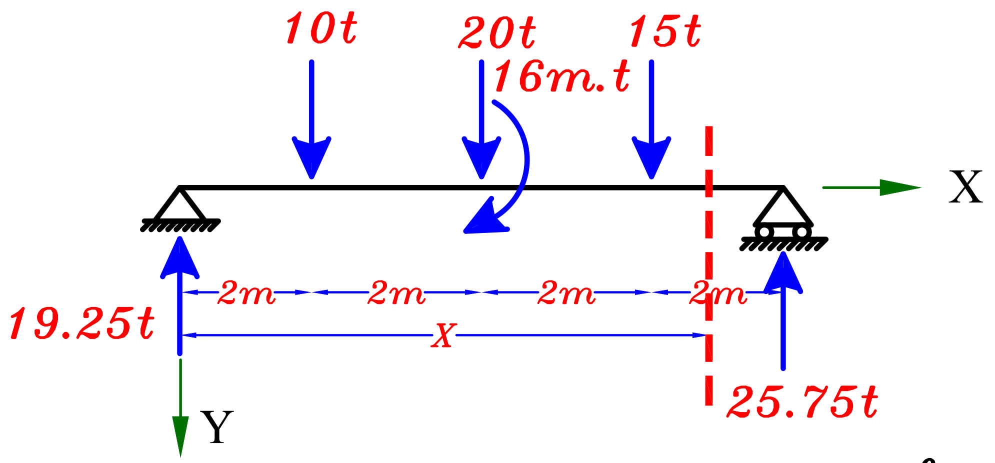
1) Concentrated moment

اذا وجد عزم مركز فى المسألة لمعرفة ما اذا كان سيدخل معنا ام لا عند اى قطاع يتم ضرب العزوم فى مسافة (قوس) و اذا كان ما بداخل القوس سالب يتم اهماله اما اذا كان موجب يتم اخذه فى الحسابات و حيث انه من الخطأ ضرب العزم فى مسافة يتم تصحيح الخطأ بجعل مسافة العزم المركز فى قوس اس صفر



$$M(x)=19.25(x)-10(x-2)-20(x-4)+16(x-4)^{0}-15(x-6)$$

وضع مسافة ال *moment* فى قوس أس صفر يجعل قيمته عند حساب ال *moment* تساوى واحد لان أى رقم أس صفر يساوى واحد و بالتالى كأننا لم نضرب ال *moment* فى مسافة

2) Uniform Load

اذا وجد *uniform load* على الكمرة يجب ان يكون القطاع يمر بال *uniform load* و لحل المشكلة سندرس كل الاحتمالات الممكنة

a) Uniform Load over the total length of beam

اذا كان ال *uniform load* على طول الكمرة لا توجد مشكلة و سيكون القطاع يحقق الكمرة كلها و يقطع ال *uniform load* .

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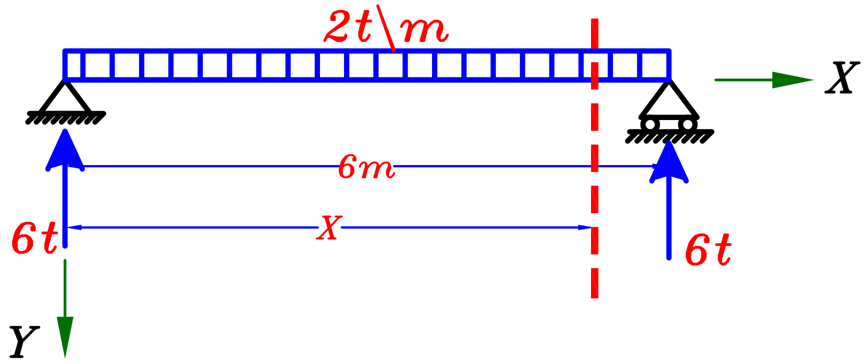
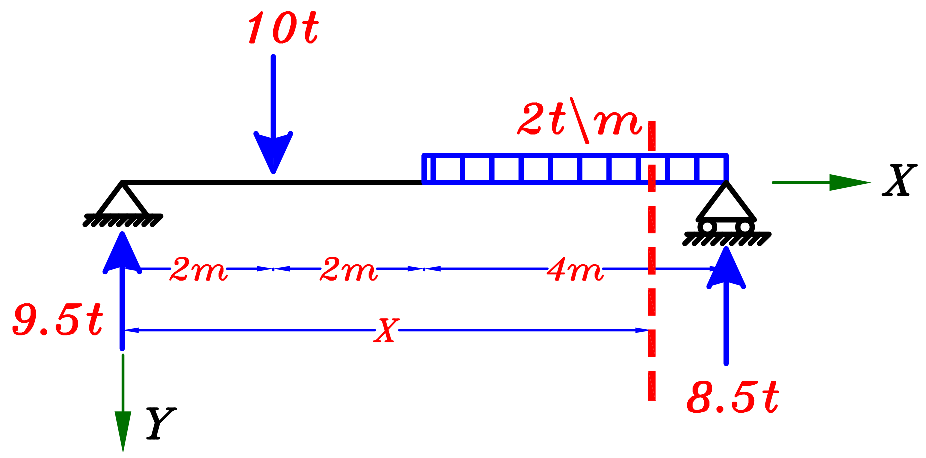


$$M(x) = 6(x) - 2(x)\ (x/2)$$

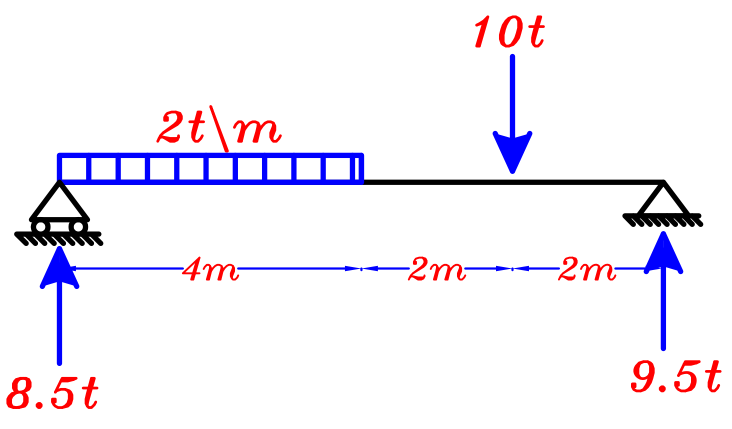
b) Uniform Load from the right side of beam



اذا كان الـ *uniform load* على اول الكمرة من ناحية اليمين لاتوجد مشكلة و نأخذ المحاور كالمعتاد و سيكون القطاع يحقق الكمرة كلها و يقطع الـ *uniform load*.

$$M(x) = 9.5(x) - 10(x-2) - 2(x-4)(x-4)/2$$

C) Uniform Load from the left side of beam



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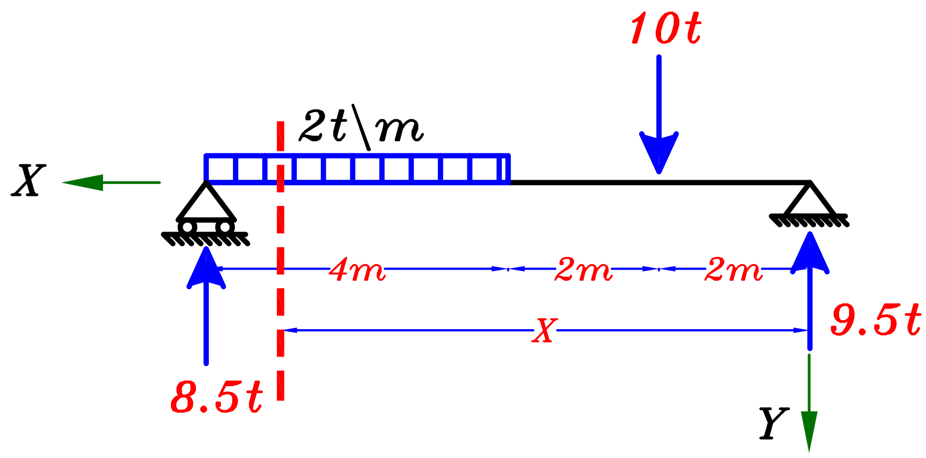
اذا كان الـ*uniform load* على اول الكمرة من ناحية اليسار نقوم بعكس المحاور و نأخذ القطاع من الناحية اليسرى و سيكون القطاع يحقق الكمرة كلها و يقطع الـ *uniform load* .



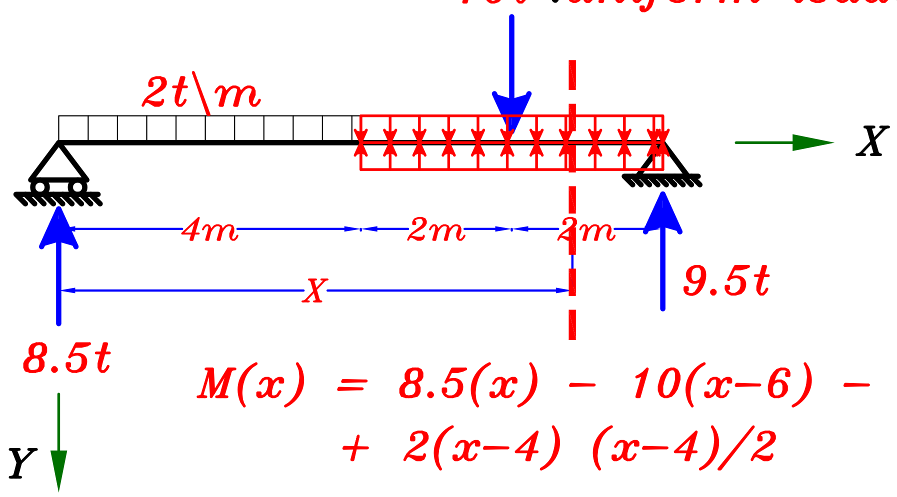
$$M(x) = 9.5(x) - 10(x-2) - 2(x-4)(x-4)/2$$

حل أخر

نفرض محور (X) لليمين و محور (Y) لاسفل .

يفضل استخدام هذا الحل .

يتم زيادة الـ*uniform load* على الكمرة من ناحية القطاع حتى يقطعه و لتصحيح هذه الز يادة يتم طرح الـ *uniform load* من نفس المكان الذى تم زيادته فى اتجاه معاكس و سيكون القطاع يحقق الكمرة كلها و يقطع الـ*uniform load*.



$$M(x) = 8.5(x) - 10(x-6) - 2(x)\ (x/2) + 2(x-4)\ (x-4)/2$$

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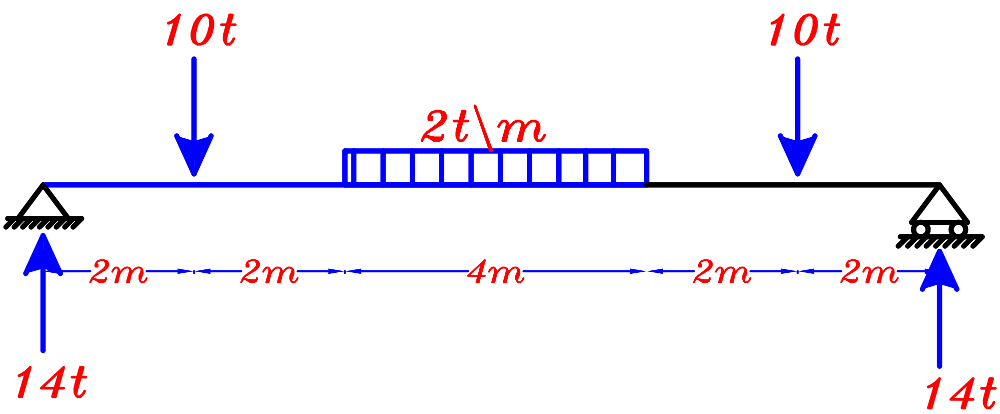
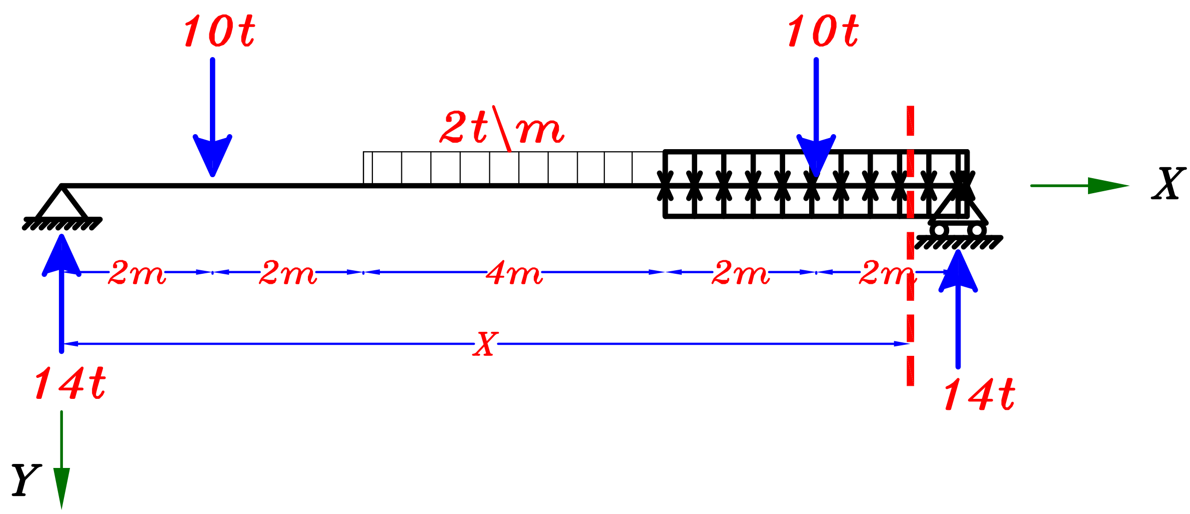
d) Uniform Load at the middle of beam

اذا كان ال uniform load فى منتصف الكمرة من ناحية اليسار نأخذ المحاور كالمعتاد و يتم زيادة ال uniform load على الكمرة من ناحية القطاع حتى يقطعه و لتصحيح هذه الز يادة يتم طرح ال uniform load من نفس المكان الذى تم زيادته فى اتجاه معاكس و سيكون القطاع يحقق الكمرة كلها و يقطع ال uniform load.





$$M(x) = 14(x) - 10(x-2) - 2(x-4)\ (x-4)\ /\ 2$$
$$+\ 2(x-8)\ (x-8)/2$$

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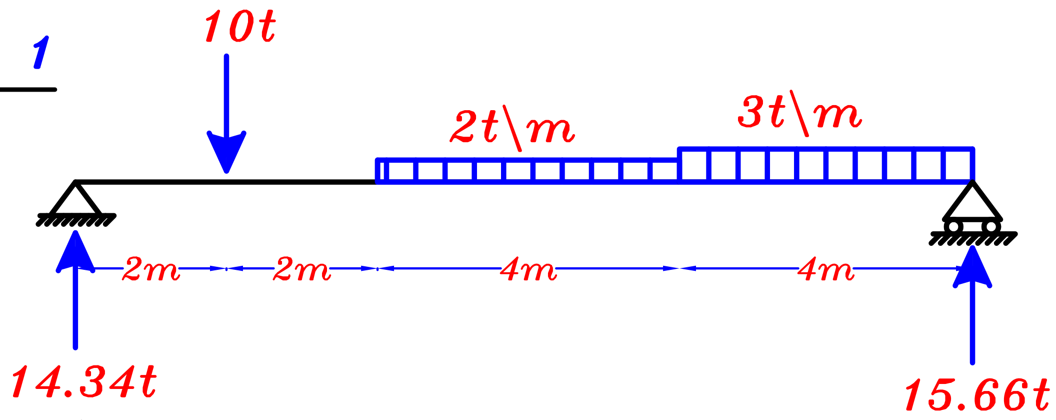
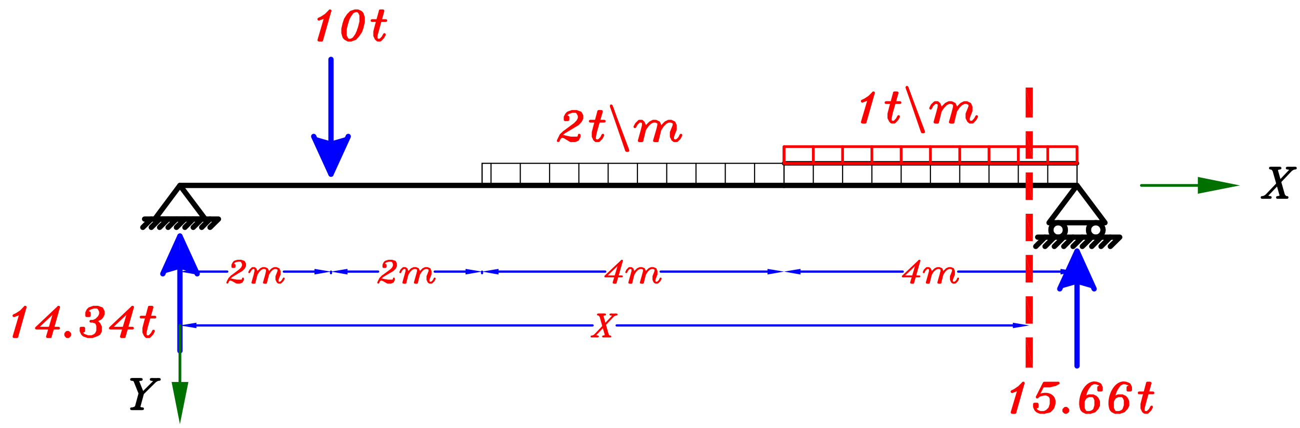
e) Uniform Load has a variable value

Case 1

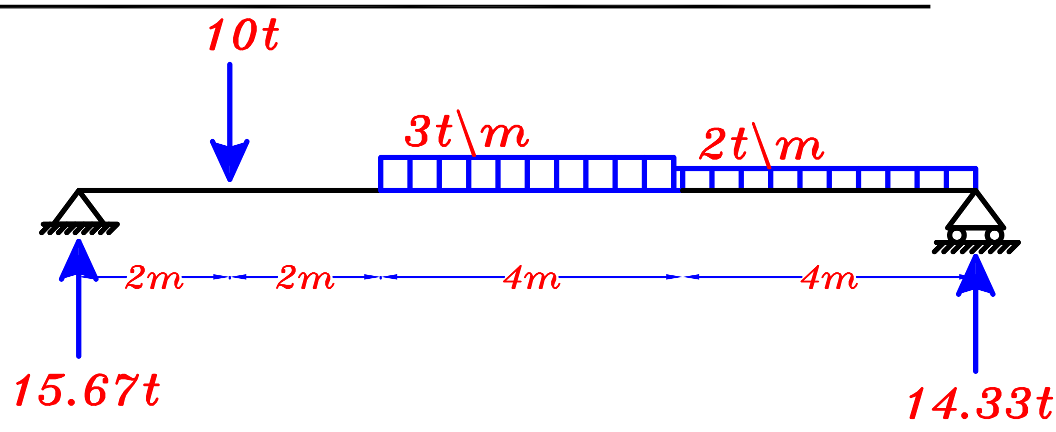


اذا كان ال *uniform load* ذو قيمة متغيرة نقوم بتقسيمه كالتالى .

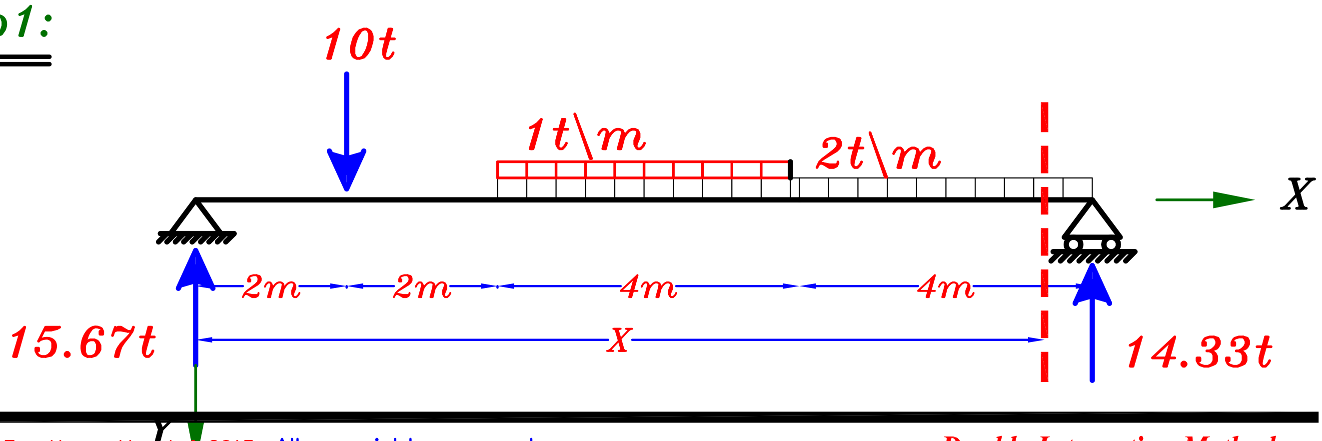


$$M(x) = 14.34(x) - 10(x-2) - 2(x-4)\,(x-4)\,/\,2$$
$$- 1(x-8)\,(x-8)\,/\,2$$

Case 2



Step1:



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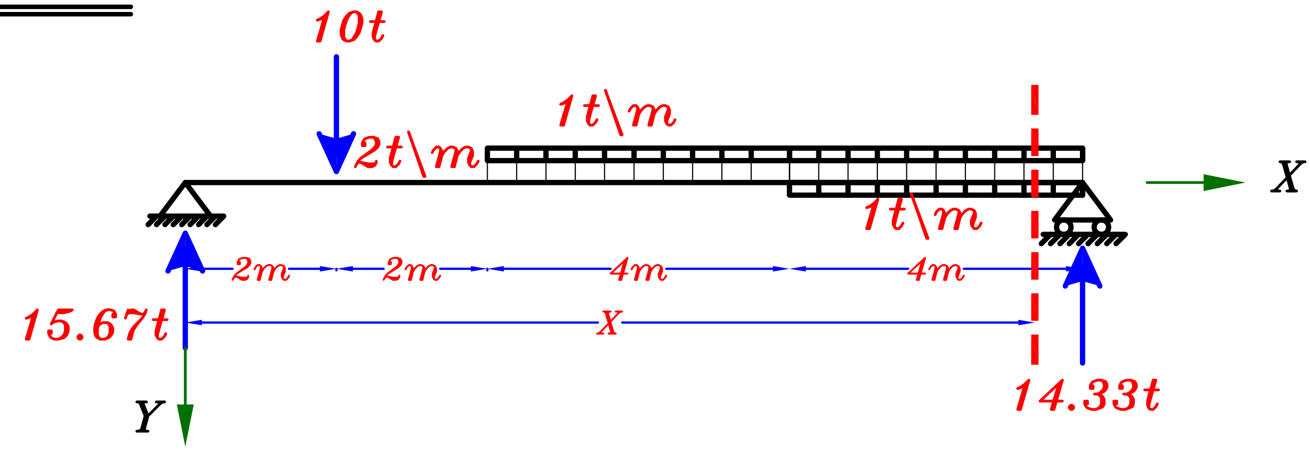
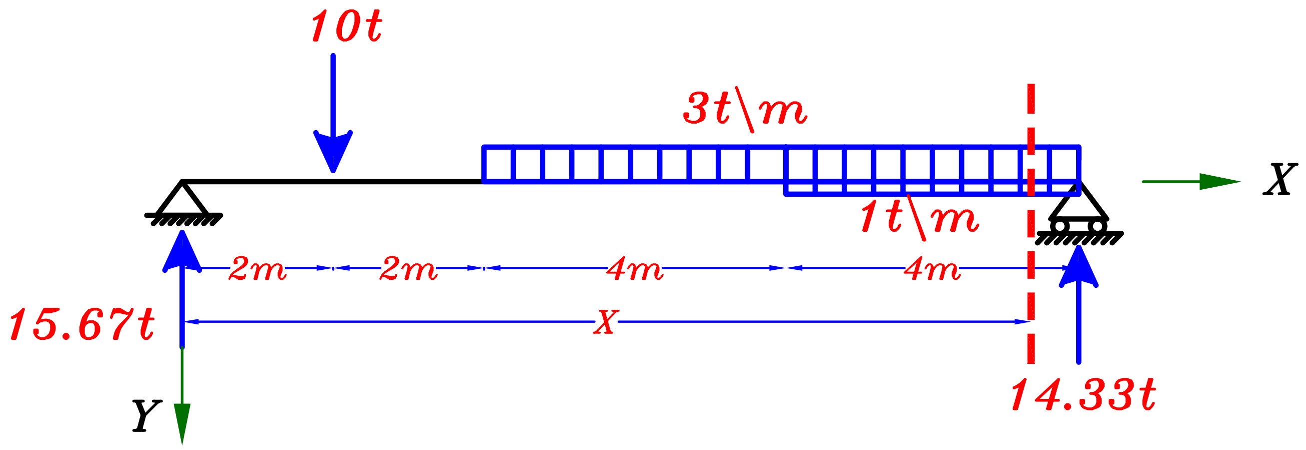
Step2:



Step3:



$$M(x) = 15.67(x) - 10(x-2) - 3(x-4)\ (x-4)\ /\ 2$$

$$+\ 1(x-8)\ (x-8)\ /\ 2$$

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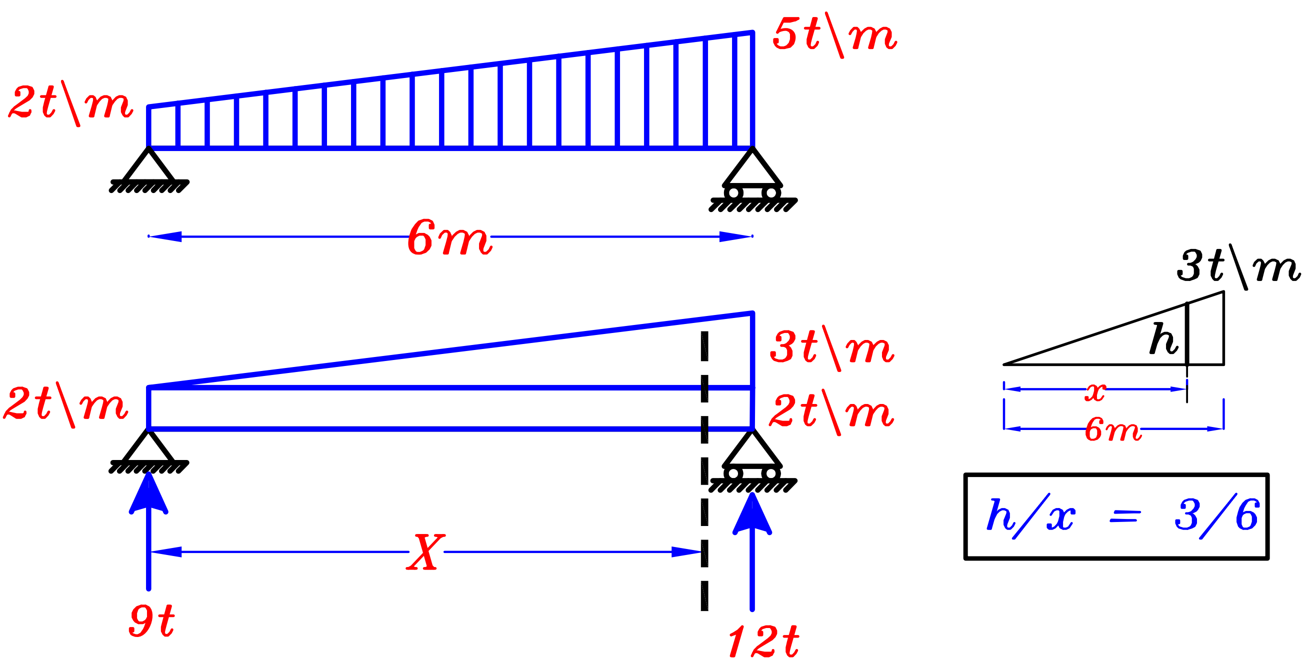
3) Nonuniform Load

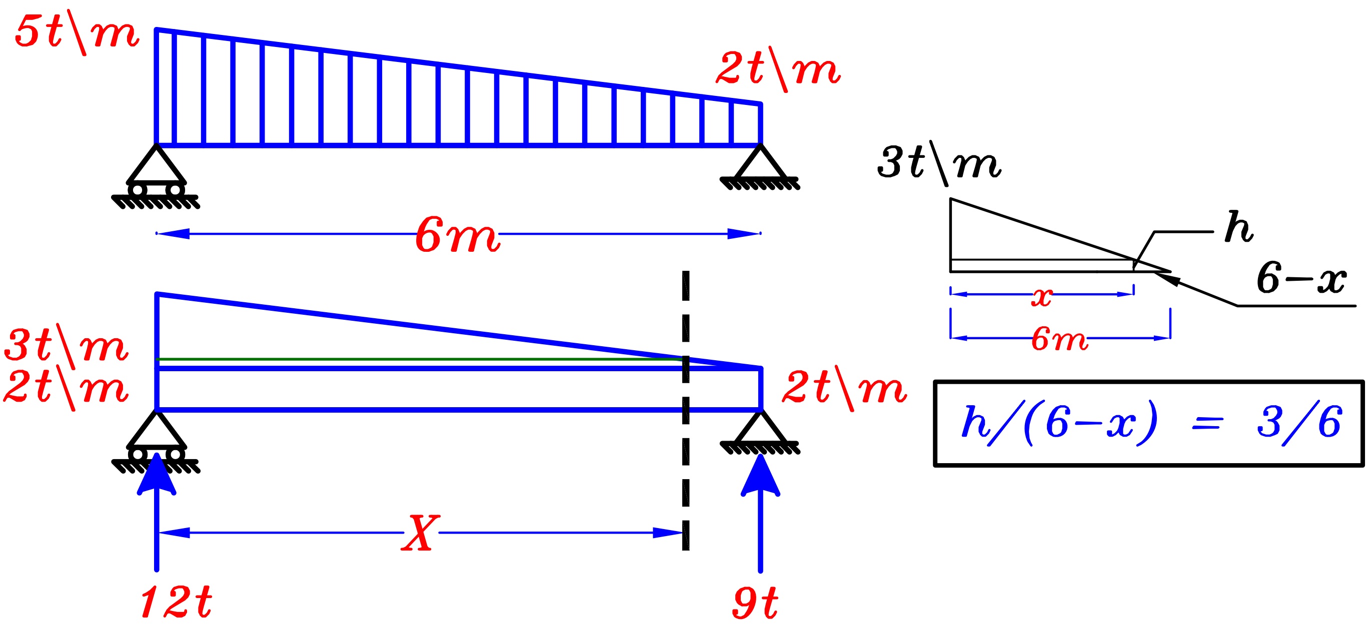
CASE (1)



$$\boxed{h/x = 3/6}$$

$$M(x) = 9(x) - 2(x)\ (x/2) - \tfrac{1}{2}\left(\tfrac{3}{6}x\right)(x)\ (x/3)$$

CASE (2)



$$\boxed{h/(6-x) = 3/6}$$

$$M(x)=12(x) - 2(x)\ (x/2) - \left[\tfrac{3}{6}(6-x)\ (x)\right](x/2)$$
$$-\tfrac{1}{2}(x)\ (3-h)\ (2x/3)$$

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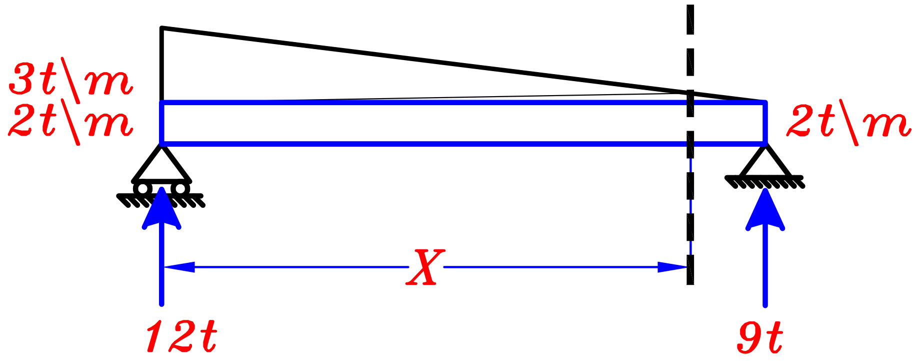
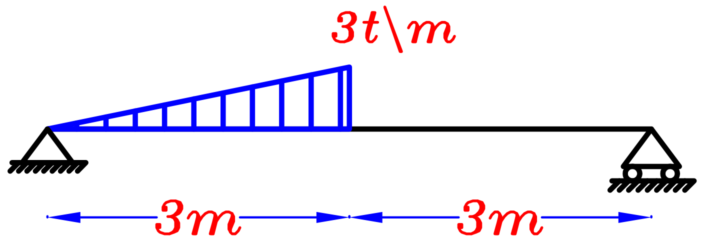
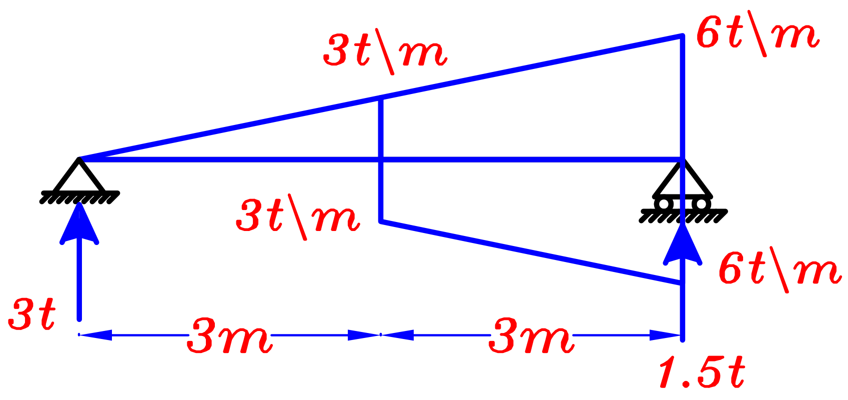
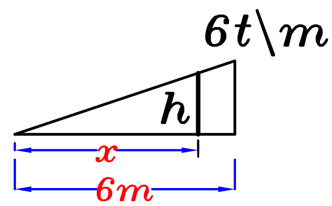


$$M(x) = 12(x) - 2(x)\,(x/2) - [\,\tfrac{1}{2}\ x\ 3\ (x)\,]\,(2x/3) - \tfrac{1}{2}\,(h)\,(x)\,(x/3)$$

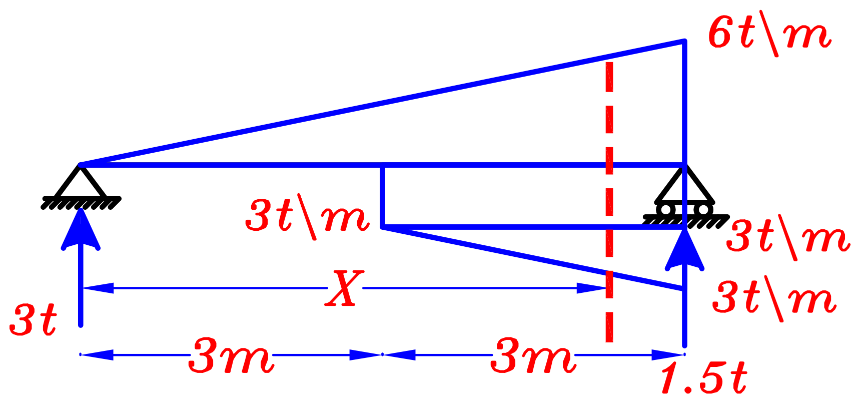
CASE (3)

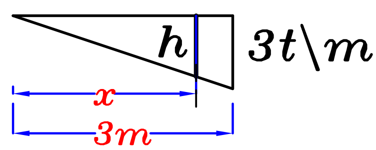






$h/x = 6/6$





$h/x = 3/3$

$$M(x) = 3(x) - \tfrac{1}{2}\,(x)\,(\tfrac{6}{6}\,x)\,(x/3) + (3)\,(x-3)\,(x-3)/2 + \tfrac{1}{2}\,(x-3)\,(\tfrac{3}{3}\ x\ )\,(x-3)/3$$

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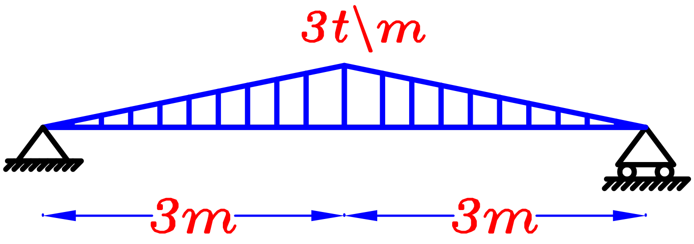
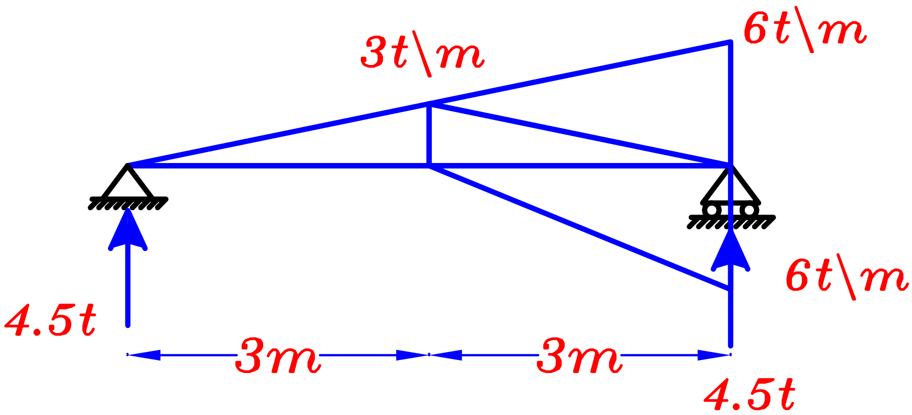
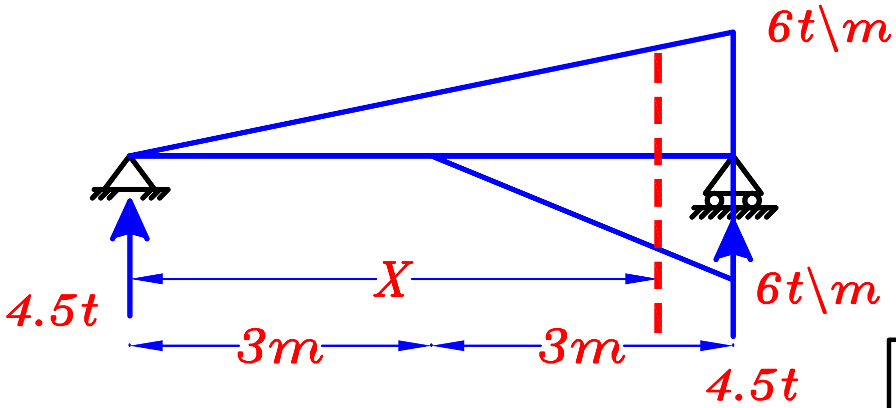
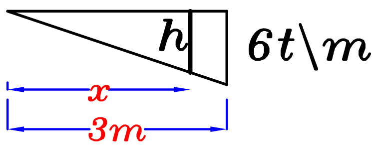
CASE (4)









$$\boxed{h/x = 6/3}$$

$$M(x) = 4.5(x) - \tfrac{1}{2}\,(x)\,\left(\tfrac{6}{6}\,x\right)\,(x/3)$$

$$+ \tfrac{1}{2}\,(x-3)\,\left(\tfrac{6}{3}\,x\right)\,(x-3)/3$$

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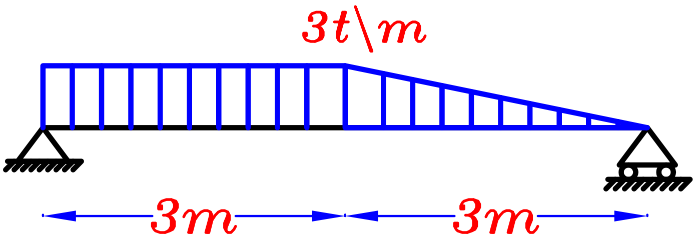
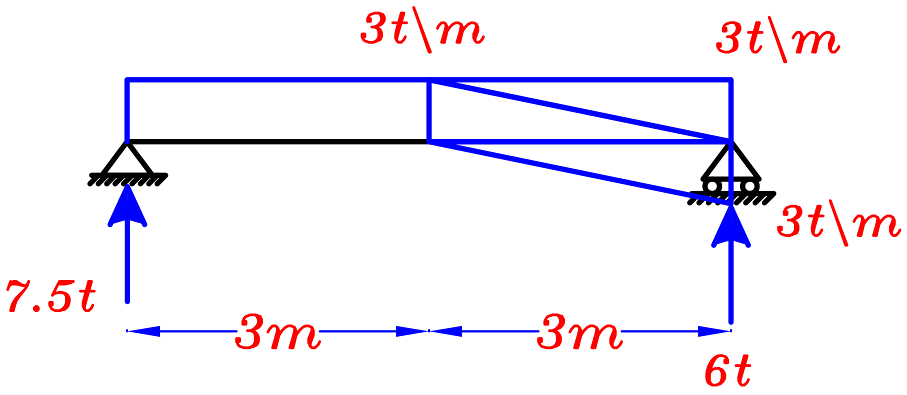
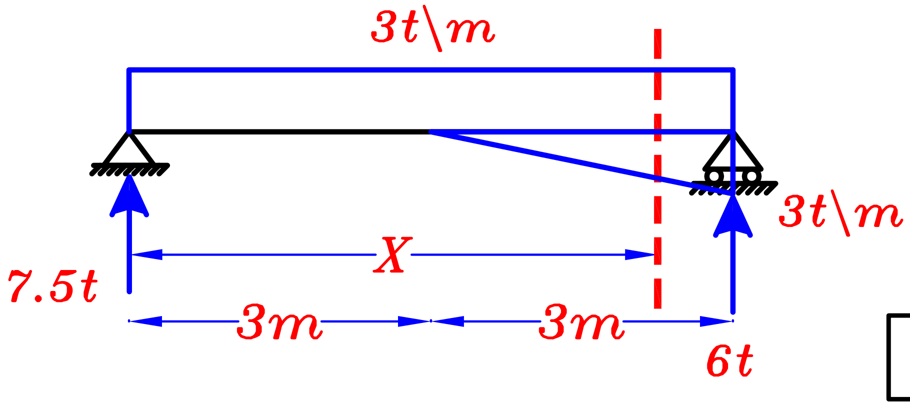
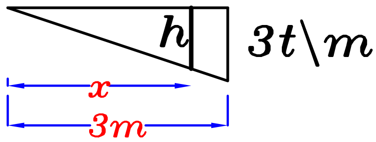
CASE (4)









$$\boxed{h/x = 3/3}$$

$$M(x) = 7.5(x) - 3\,(x)\,(x/2)$$

$$+\ \tfrac{1}{2}\,(x-3)\,(\tfrac{3}{3}\,x)\,(x-3)/3$$

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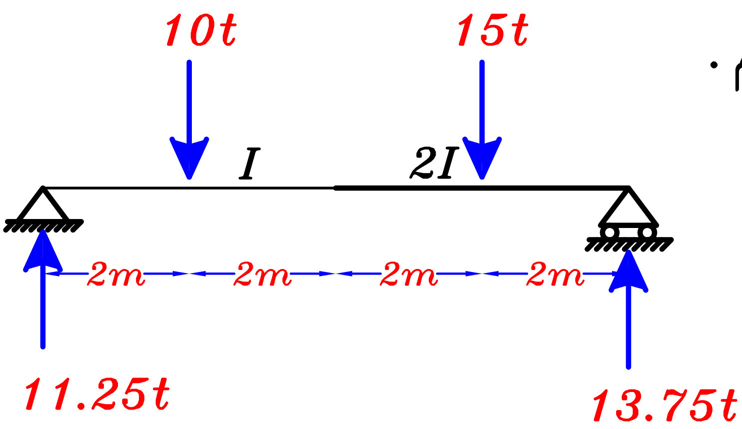
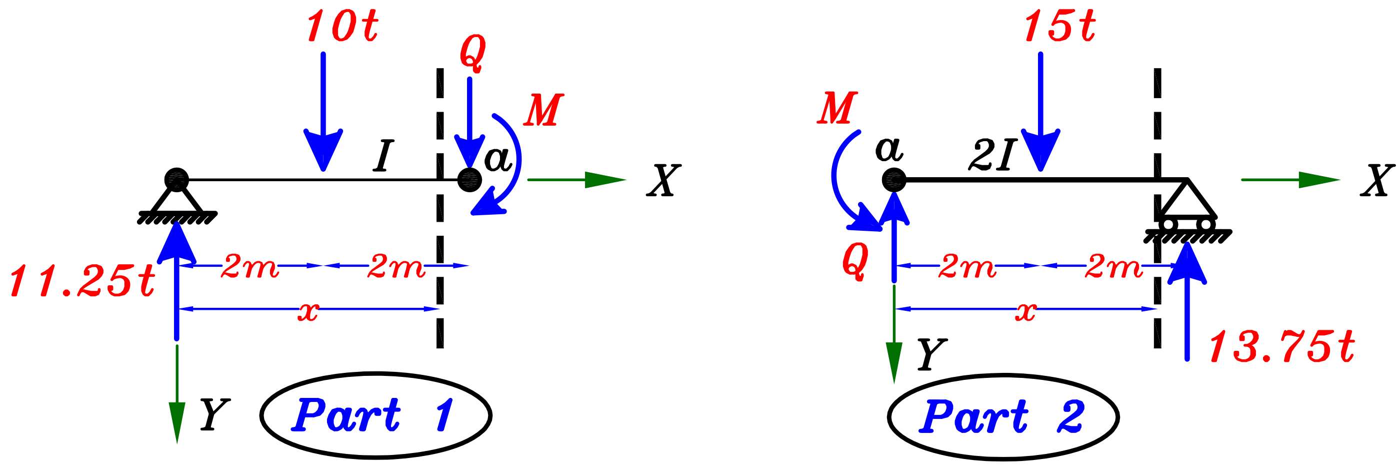
4) Change in beam inertia

اذا كان قطاع الكمرة متغير و المعادلة التفاضلية مستنتجة على أساس ان القطاع ثابت نقوم بفصل المسألة عند نقطة الاختلاف فى الـ *Inertia* و يتم الربط بين الجزئين على اساس ان الـ *deflection* والـ *slope angle* عند نقطة التقسيم .



عند نقطة التقسيم يظهر ثلاثة مجاهيل و هم الـ *X,Y,M* .



For part 1

$$M\,(x) = 11.25(x) - 10(x-2)$$

$$EIy'' = 10\,(x-2) - 11.25(x)$$

$$EIy' = 5\,(x-2)^2 - (11.25/2)\,(x)^2 + C_1$$

$$EIy = (5/3)\,(x-2)^3 - (11.25/6)\,(x)^3 + C_1 x + C_2$$

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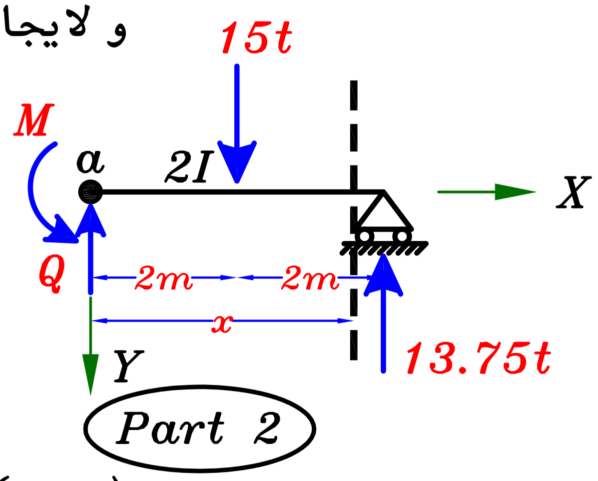
For part 2

$$M(x) = -M(x)^0 + Q(x) - 15(x-2)$$

$$2EIy'' = M(x)^0 - Q(x) + 15(x-2)$$

و لايجاد قيمة الQ,M عند نقطة a عند نقطة نطبق
معادلات الاتزان على أى من الجزئين



$\Sigma Y = 0 \Longrightarrow$ $Q=1.25t$

$\Sigma M@a = 0 \Longrightarrow$ $M=-25m.t$

$$2EIy'' = -25(x)^0 - 1.25(x) + 15(x-2)$$

$$EIy'' = -12.5(x)^0 - 0.625(x) + 7.5(x-2)$$

$$EIy' = -12.5(x)^1 - 0.3125(x)^2 + 3.75(x-2)^2 + C_3$$

$$EIy = -6.25(x)^2 - 0.1042(x)^3 + 1.25(x-2)^3 + C_3x + C_4$$

To get C1 , C2 , C3 ,and C4

For part 1

at $X = 0$ m $\Longrightarrow$ $Y=0$ نعوض فى معادلة الdeflectionل Part 1

$$EIy = 0 = (5/3)(0-2)^3 - (11.25/6)(0)^3 + C_1 0 + C_2$$

$$\boxed{C_2 = 0}$$

For part 2

at $X = 4$ m $\Longrightarrow$ $Y=0$ نعوض فى معادلة الdeflectionل Part 2

$$EIy=0=-6.25(4)^2 - 0.1042(4)^3 + 1.25(4-2)^3 + C_3 4 + C_4$$

$$\boxed{96.67 = 4C_3 + C_4 \Longrightarrow Eq.(1)}$$

Y_a from part 1 $= Y_a$ from part 2

Y_a from part 1 $(@x=4)$

$$= (5/3)(4-2)^3 - (11.25/6)(4)^3 + C_1 4 + C_2$$

$$= 4C_1 + C_2 - 106.667$$

$$= 4C_1 + 0 - 106.667 = 4C_1 - 106.667$$

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Y_a from part 2 (@x=0)

$$= -\ 6.25\ (0)^2 - 0.1042(0)^3 + 1.25(\cancel{0-2})^3 + C_3 0 + C_4$$

–ve

$$= C_4$$

Y_a from part 1 $= Y_a$ from part 2

$$4C_1 - 106.667 = C_4$$

$$\boxed{4C_1 - C_4 = 106.667 \Longrightarrow Eq.(2)}$$

$Y\grave{}_a$ from part 1 $= Y\grave{}_a$ from part 2

$Y\grave{}_a$ from part 1 (@x=4)

$$= 5\ (4-2)^2 - (11.25/2)\ (4)^2 + C_1 = -70 + C_1$$

$Y\grave{}_a$ from part 2 (@x=0)

$$= -12.5\ (0)^1 - 0.3125(0)^2 + 3.75(\cancel{0-2})^2 + C_3 = C_3$$

–ve

$Y\grave{}_a$ from part 1 $= Y\grave{}_a$ from part 2

$$-70 + C_1 = C_3$$

$$\boxed{C_1 - C_3 = 70 \Longrightarrow Eq.(3)}$$

Solving equations 1,2,3

$\boxed{C_1 = 60.42}$ $\boxed{C_3 = -9.58}$ $\boxed{C_4 = 135}$

For part 1

$$EIy\grave{} = 5\ (x-2)^2 - (11.25/2)\ (x)^2 + 60.42$$

$$EIy = (5/3)\ (x-2)^3 - (11.25/6)\ (x)^3 + 60.42x$$

For part 2

$$EI\,y\grave{} = -6.25\ (x)^2 - 0.3125(x)^2 + 3.75(x-2)^2 - 9.58$$

$$EI\,y = -2.083\ (x)^3 - 0.1042(x)^3 + 1.25(x-2)^3 - 9.58\ x + 135$$

ملحوظة هامة جدا

يفضل الحل بفرض محور (X) لليمين

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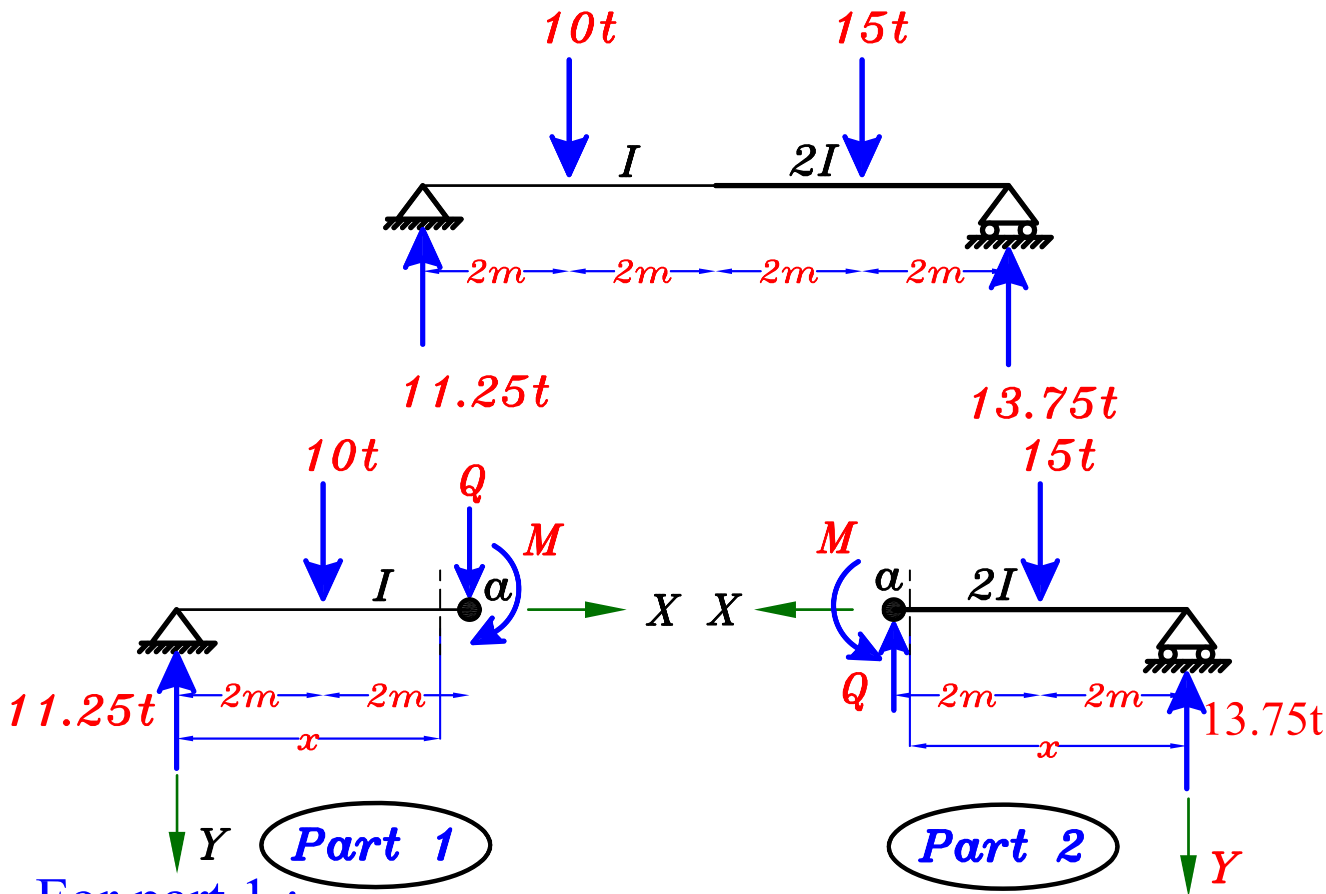
طريقة أخرى للحل

و هى أن نفرض فى الجزء الاول محور X لليمين و فى الجزء الثانى نفرض محور X لليسار .

و لكن فى هذه الحالة عند مساوات الـ *Slope angle* عند نقطة a من اليمين و اليسار تكون الاشارة معكوسة و ذلك لان المحاور معكوسة و لكن فى الـ *deflection* لا نعكس الاشارات .

$$\boxed{\begin{array}{c} Y'_a \text{ from part 1} = -\,Y'_a \text{ from part 2} \\ Y_a \text{ from part 1} = Y_a \text{ from part 2} \end{array}}$$



For part 1 :

M (x) = 11.25 (x) - 10 (x - 2)

$EIy'' = 10 (x - 2) - 11.25 (x)$

$EIy' = 5 (x - 2)^2 - (11.25 / 2) (x)^2 + C1$

$EIy = (5 / 3) (x - 2)^3 - (11.25 / 6) (x)^3 + C1 x + C2$

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For part 2 :

$$M(x) = 13.75\,(x) - 15\,(x-2)$$

$$2EIy'' = 15\,(x-2) - 13.75\,(x)$$

$$2EIy' = (15/2)\,(x-2)^2 - (13.75/2)\,(x)^2 + C3$$

$$2EIy = (15/6)\,(x-2)^3 - (13.75/6)\,(x)^3 + C3\,x + C4$$

To get C1 , C2 , C3 ,AND C4

For part 1 :

At $x = 0$ m ----------------- $y = 0$

$$\boxed{C2 = 0}$$

At $x = 4$ m ------------------ $EIy_a = 4C1 - 106.67$ & $EIy'_a = -70 + C1$

For part 2 :

At $x = 0$ m ------------------ $y = 0$

$$\boxed{C4 = 0}$$

At $x = 4$ m ------------------$2EIy_a = 4C3 - 126.66$ & $2EIy_a = -80 + C3$

To get (C1&C3)

y_a from part 1 = y_a from part 2

y'_a from part 1 = $-y'_a$ from part 2

$$\boxed{C1 = 60.375} \qquad \boxed{C3 = 99.25}$$

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For part 1 :

$$EIy' = 5 (x - 2)^2 - (11.25 / 2) (x)^2 + 60.375$$

$$EIy = (5 / 3) (x - 2)^3 - (11.25 / 6) (x)^3 +60.375 (x)$$

For part 2 :

$$2EIy' = (15 / 2) (x - 2)^2 - (13.75 / 2) (x)^2 + 99.25$$

$$2EIy = (15 / 6) (x - 2)^3 - (13.75 / 6) (x)^3 + 99.25 (x)$$

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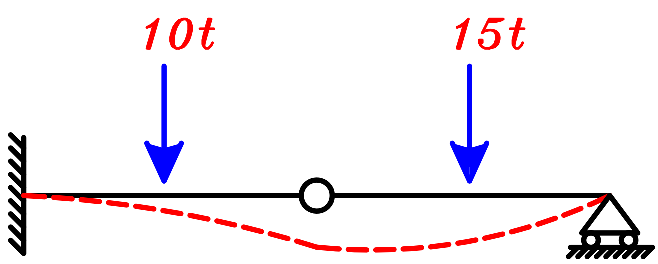
5) Intermediate hinge

فى حالة وجود كمرة بها *Intermediate Hinge* يجب تقسيم الكمرة من عند الـ *Intermediate Hinge* و حل الكمرة على انها مسألتين و لفهم ذلك نقوم برسم الـ *Elastic Line* .



المعادلة التى يتم استنتاجها ما هى الا معادلة الـ *Elastic Line* و هى $EIy = P1\ (X) + P2\ (X-L) + ---- + C1\ (X) + C2$

و الـ *Elastic Line* فى هذه الحالة معادلتين مختلفتين و لذلك لا يمكن كتابة معادلة واحدة لحساب الـ *deflection* و لذلك نحل كل جزأ على حده ولايجاد ثوابت التكامل نساوى الـ *deflection* عند نقطة الفصل مع الاخذ فى الاعتبار ان الـ *slope angle* عند نقطة الفصل غير متساوى و ذ لك لاختلاف معادلة الـ *deflection* لكل كمرة و لذلك فى هذه المسائل لا نحسب الـ *slope angle* عند الـ *Intermediate Hinge* و لكن نحسب الـ *change inslope angle* عن طريق حساب الـ *slope angle* من كل جهة وايجاد الفرق بينهما .

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خطوات حل المسائل :

١- يتم تحديد الـ *Section*.

٢- نكتب معادلة الـ *Moment* عند هذا الـ *Section* .

٣- نكتب معادلة Y'' عن طريق ضرب معادلة الـ *Moment* فى $-Ve$.

٤- نكامل معادلة الـ Y'' فنحصل على Y' .

٥- نكامل معادلة الـ Y' فنحصل على Y .

٦- عن طريق الـ *Boundary Conditions* نحصل على الثوابت .

٧- بالتعويض فى معادلة الـ Y' أو الـ Y بقيمة الـ X لاى نقطة نحصل على الـ(Y') *Slope angle* أو الـ *Deflection* (Y) .

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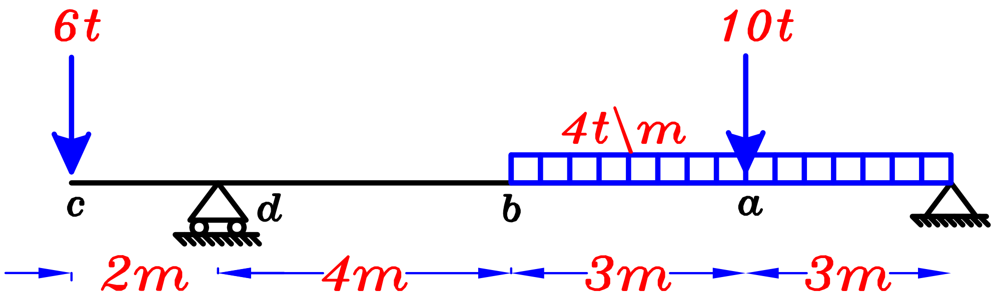
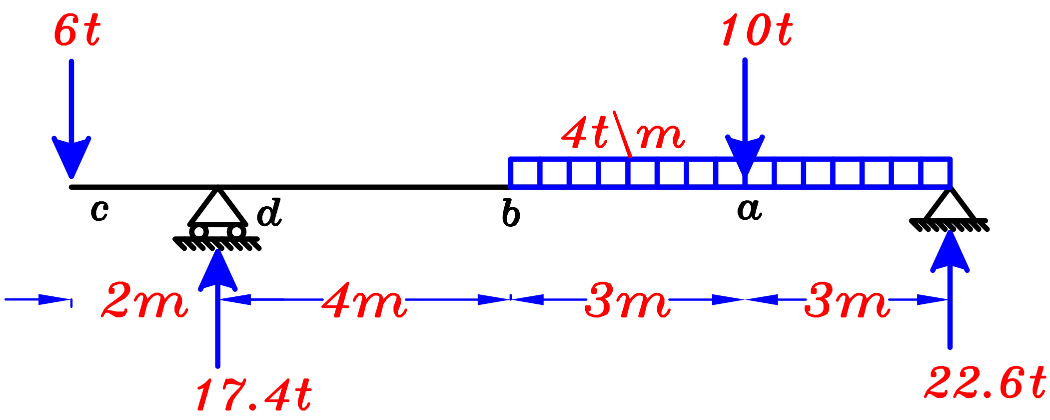
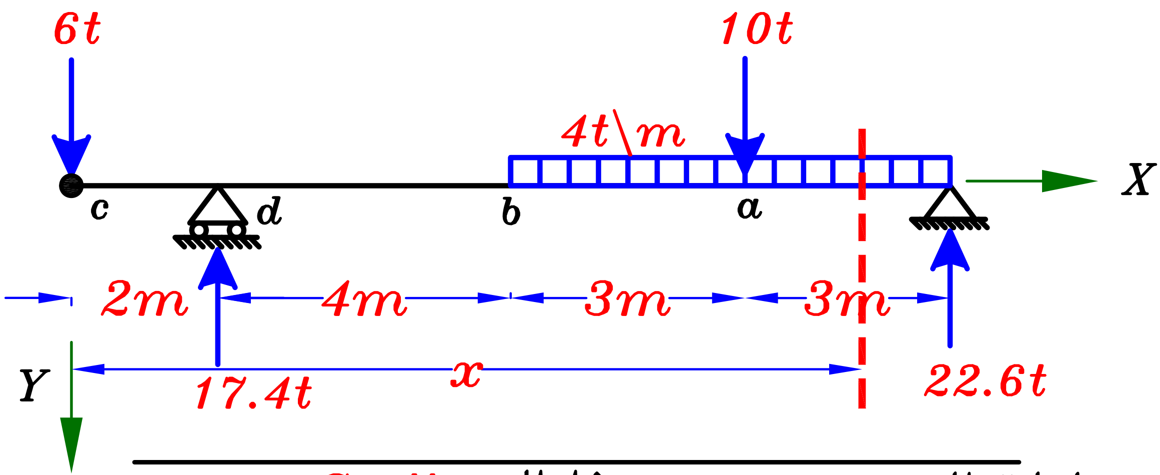
Example:

For the shown beam find the deflection at points (a&b&c) and the slope angle at points (c & d) . applying double integration method . ($EI = 20000\ m^2.t.$)





١- يتم تحديد الـ *Section*.



٢- نكتب معادلة الـ*Moment* عند هذا الـ*Section* .

$$M(x) = 17.4(x-2) - 6(x) - 10(x-9) - 4(x-6)\ (x-6)/2$$

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٣- نكتب معادلة Y'' عن طريق ضرب معادلة الـ *Moment* في *−Ve* .

$$EIy'' = -M(x)$$

$$EIy'' = -17.4\ (x-2) + 6(x) + 10(x-9) + 4(x-6)(x-6)/2$$

٤- نكامل معادلة الـY'' فنحصل على Y' .

$$EIy' = -(17.4/2)\ (x-2)^2 + 3\ (x)^2 + 5(x-9)^2 + (2/3)\ (x-6)^3 + C_1$$

٥- نكامل معادلة الـY' فنحصل على Y .

$$EIy = -(17.4/6)\ (x-2)^3 + (x)^3 + (5/3)\ (x-9)^3 + (2/12)\ (x-6)^4 + C_1(x) + C_2$$

٦- عن طريق الـ *Boundary Conditions* نحصل على الثوابت .

From intial conditions

at $X = 2\ m \Longrightarrow Y=0 \Longrightarrow$ نعوض بها فى معادلة الـ *deflection*

at $X = 12\ m \Longrightarrow Y=0 \Longrightarrow$ نعوض بها فى معادلة الـ *deflection*

$$EIy = -(17.4/6)\ (x-2)^3 + (x)^3 + (5/3)\ (x-9)^3 + (2/12)\ (x-6)^4 + C_1(x) + C_2$$

AT X = 2

$$EIy = 0 = -(17.4/6)\ (2-2)^3 + (2)^3 + (5/3)\ \overset{-ve}{\cancel{(2-9)}}^{\,3} + (2/12)\ \underset{-ve}{\cancel{(2-6)}}^{\,4} + C_1(2) + C_2$$

$$2C_1 + C_2 = -8 \Longrightarrow Eq.(1)$$

AT X = 12

$$EIy = 0 = -(17.4/6)\ (12-2)^3 + (12)^3 + (5/3)\ (12-9)^3 + (2/12)\ (12-6)^4 + C_1(12) + C_2$$

$$12C_1 + C_2 = 911 \Longrightarrow Eq.(2)$$

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Solving equations 1,2,3

$$\boxed{C_1 = 91.9} \qquad \boxed{C_2 = -191.8}$$

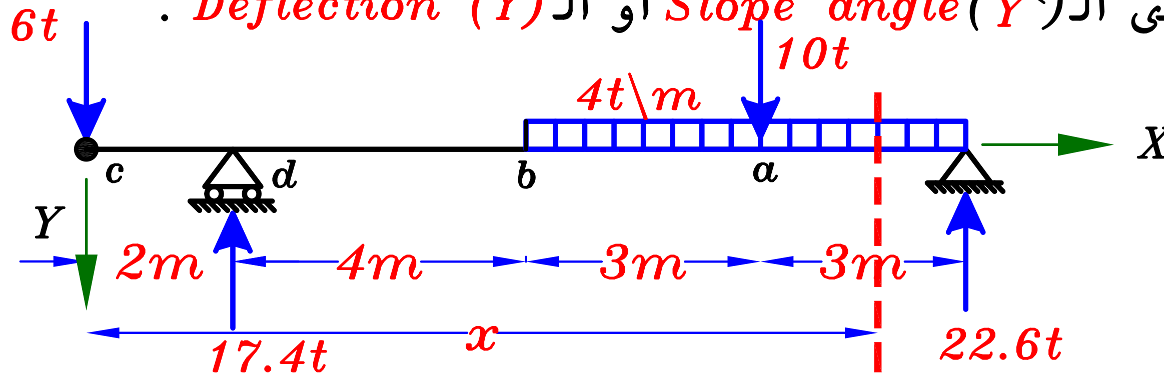
$$EIy' = -(17.4/2)\ (x-2)^2 + 3\ (x)^2 + 5(x-9)^2 + (2/3)\ (x-6)^3 + 91.9 \Longrightarrow$$ معادلة ال Slope angle

$$EIy = -\ (17.4/6)\ (x-2)^3 + (x)^3 + (5/3)\ (x-9)^3 + (2/12)\ (x-6)^4 + 91.9\ (x) - 191.8 \Longrightarrow$$ معادلة ال deflection

---

٧- بالتعويض فى معادلة الY' أو ال Y بقيمة الX لاى نقطة

نحصل على ال(Y') Slope angle أو ال(Y) Deflection .



At point (a)

لحساب ال deflection نعوض فى معادلة ال deflection

at Point a $\Longrightarrow$ $X = 9\ m$

$$EIy = -\ (17.4/6)\ (9-2)^3 + (9)^3 + (5/3)\ (9-9)^3 + (2/12)\ (9-6)^4 + 91.9\ (9) - 191.8 = 383.1$$

$$y_a = \frac{383.1}{EI} = \frac{383.1}{20000} = 0.0191m = 1.91\ cm$$

At point (b)

لحساب ال deflection نعوض فى معادلة ال deflection

at Point b $\Longrightarrow$ $X = 6\ m$

$$EIy = -\ (17.4/6)\ (6-2)^3 + (6)^3 + (5/3)\ \cancel{(6-9)^3}^{-ve} + (2/12)\ (6-6)^4 + 91.9\ (6) - 191.8 = 383.1$$

$$y_b = \frac{396}{EI} = \frac{396}{20000} = 0.0198m = 1.98\ cm$$

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At point (C)

لحساب الـ *deflection* نعوض فى معادلة الـ *deflection*

at Point C $\Longrightarrow$ $X = 0\ m$

$$EIy = -(17.4/6)\ (0\overset{-ve}{\cancel{-}}2)^3 + (0)^3 + (5/3)\ (0\overset{-ve}{\cancel{-}}9)^3$$
$$+ (2/12)\ (0\underset{-ve}{\cancel{-}}6)^4 + 91.9\ (0) - 191.8 = 383.1$$

$$y_c = \frac{-191.8}{EI} = \frac{-191.8}{20000} = -0.0096m = -0.96\ cm$$

لحساب الـ *Slope angle* نعوض فى معادلة الـ *Slope angle*

at Point C $\Longrightarrow$ $X = 0\ m$

$$EIy' = -(17.4/2)\ (0\overset{-ve}{\cancel{-}}2)^2 + 3\ (0)^2 + 5(0\overset{-ve}{\cancel{-}}9)^2$$
$$+ (2/3)\ (0\underset{-ve}{\cancel{-}}6)^3 + 91.9$$

$$y'_c = \frac{91.9}{EI} = \frac{91.9}{20000} = 0.0046\ rad.$$

At point (d)

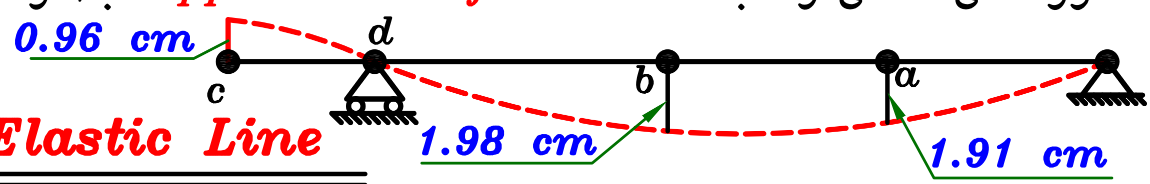
لحساب الـ *Slope angle* نعوض فى معادلة الـ *Slope angle*

at Point d $\Longrightarrow$ $X = 2\ m$

$$EIy' = -(17.4/2)\ (2-2)^2 + 3\ (2)^2 + 5(2\overset{-ve}{\cancel{-}}9)^2$$
$$+ (2/3)\ (2\underset{-ve}{\cancel{-}}6)^3 + 91.9$$

$$y'_d = \frac{115.9}{EI} = \frac{115.9}{20000} = 0.0058\ rad.$$

لرسم الـ *Elastic Line* نرسم الكمرة و نوقع عليها قيم الـ *deflection* مع الاخذ فى الاعتبار أن الـ *deflection* الـ *+Ve* يكون فى اتجاه محور *Y* أى لاسفل و الـ *-Ve* يكون عكس محور *Y* أى لاعلى و طبعا الـ *deflection* عند الـ*Support* بصفر .



Elastic Line

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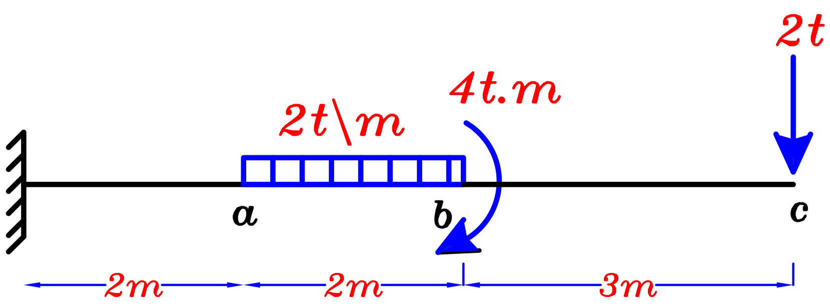
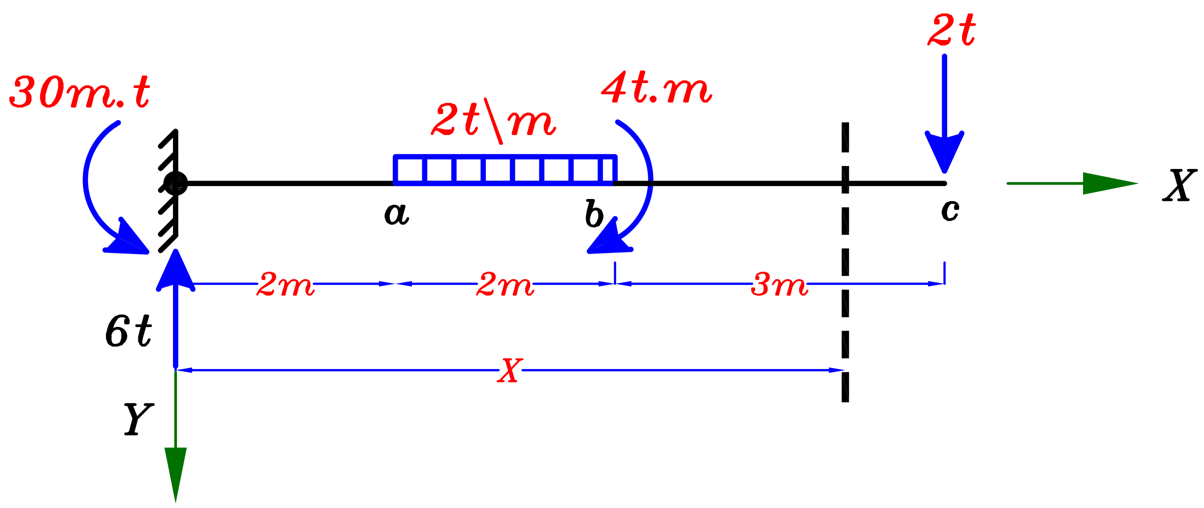
Example:

For the shown beam find the deflection at points (a&b&c) and the slope angle at points (c) . applying double integration method. (EI = 20000 m^2.t.)





$$M(x) = 6(x) - 2(x-2)\,(x-2)/2 + 4(x-4)^0 - 30(x)^0 + 2\,(x-4)\,(x-4)\,/\,2$$

$$EIy'' = -M(x)$$

$$EIy'' = -6\,(x) + 2(x-2)\,(x-2)/2 - 4(x-4)^0 + 30(x)^0 - 2\,(x-4)\,(x-4)\,/\,2$$

$$EIy' = -3\,(x)^2 + (x-2)^3/\,3 - 4(x-4) + 30(x) - (x-4)^3/\,3 + C_1$$

$$EIy = -(x)^3 + (x-2)^4/\,12 - 2(x-4)^2 + 15(x)^2 - (x-4)^4/\,12 + C_1 x + C_2$$

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From intial conditions

at $X = 0\ m \Longrightarrow Y=0 \Longrightarrow$ نعوض بها فى معادلة ال *deflection*

at $X = 0\ m \Longrightarrow Y^{`}=0 \Longrightarrow$ نعوض بها فى معادلة ال *Slope angle*

AT X = 0

$$EIy=0=-(0)^3+\underbrace{\cancel{(0-2)}^4}_{}/12-2\overset{-ve}{\cancel{(0-4)}}^2+15(0)^2$$

$$-\underset{-ve}{\cancel{(0-4)}}^4/12+C_1 0+C_2 \Longrightarrow \boxed{C_2 = 0}$$

AT X = 0

$$EIy^{`}=0=-3(0)^2+(0-2)^3/3-4(0-4)+30(0)$$

$$-(0-4)^3/3+C_1 \Longrightarrow \boxed{C_1 = 0}$$

$$EIy^{`}=-3(x)^2+(x-2)^3/3-4(x-4)+30(x)$$

$$-(x-4)^3/3 \Longrightarrow$$ معادلة ال *Slope angle*

$$EIy=-(x)^3+(x-2)^4/12-2(x-4)^2+15(x)^2$$

$$-(x-4)^4/12 \Longrightarrow$$ معادلة ال *deflection*

و بالتعويض عن قيمة X فى أى من المعادلتين يمكننا أن نحصل على قيمة ال *deflection* أو ال *Slope angle* عند أى نقطة .

At point (a)

لحساب ال *deflection* نعوض فى معادلة ال *deflection*

at Point a $\Longrightarrow X = 2\ m$

$$EIy=-(2)^3+(2-2)^4/12-2\overset{-ve}{\cancel{(2-4)}}^2+15(2)^2$$

$$-\underset{-ve}{\cancel{(2-4)}}^4/12$$

$$y_a=\frac{52}{EI}=\frac{52}{20000}=0.0026m = 0.26\ cm$$

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At point (b)

لحساب ال *deflection* نعوض فى معادلة ال *deflection*

at Point b $\Longrightarrow$ $X = 4\ m$

$$y_b = \frac{177.33}{EI} = \frac{177.33}{20000} = 0.0088m = 0.88\ cm$$

At point (C)

لحساب ال *deflection* نعوض فى معادلة ال *deflection*

at Point C $\Longrightarrow$ $X = 7\ m$

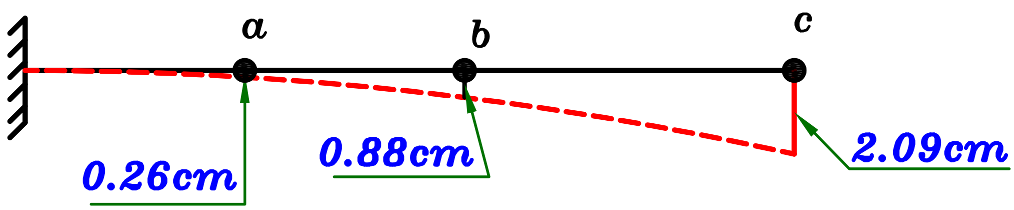
$$y_c = \frac{419}{EI} = \frac{419}{20000} = 0.02095m = 2.095\ cm$$

At point (C)

لحساب ال *Slope angle* نعوض فى معادلة ال *Slope angle*

at Point C $\Longrightarrow$ $X = 7\ m$

$$y'_c = \frac{83.66}{EI} = \frac{83.66}{20000} = 0.00418\ rad.$$



Elastic Line

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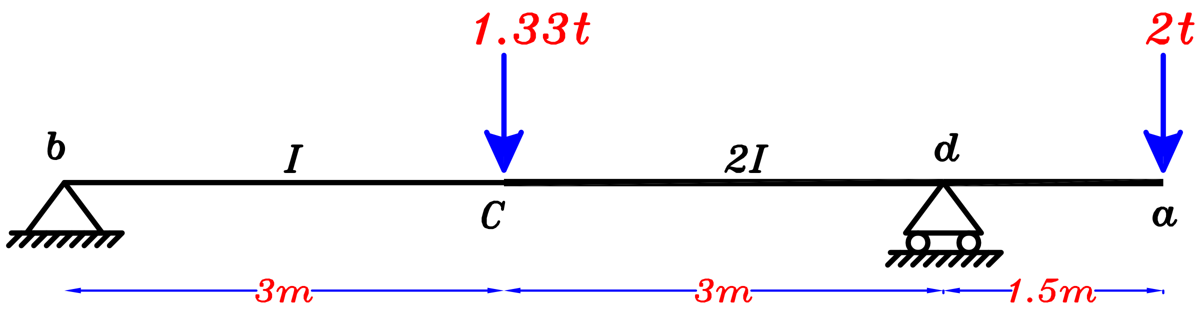
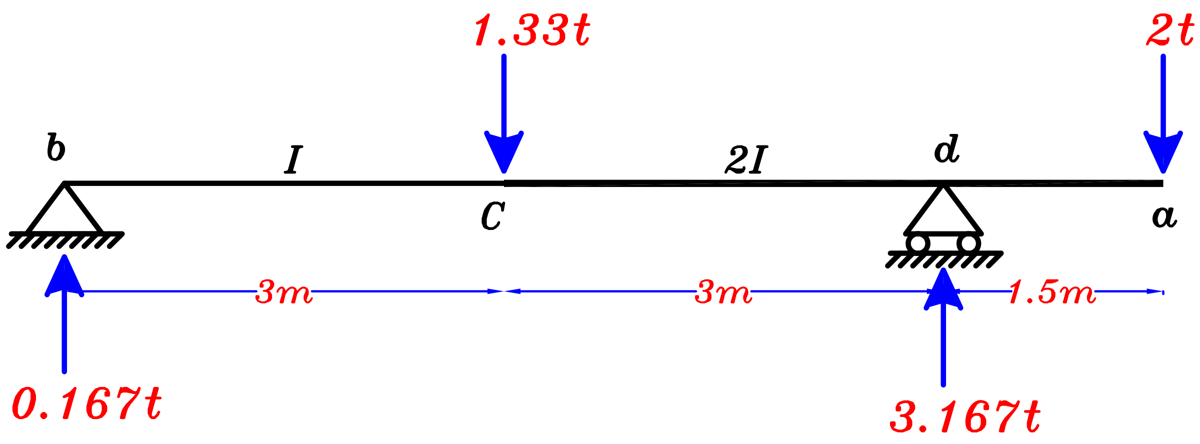
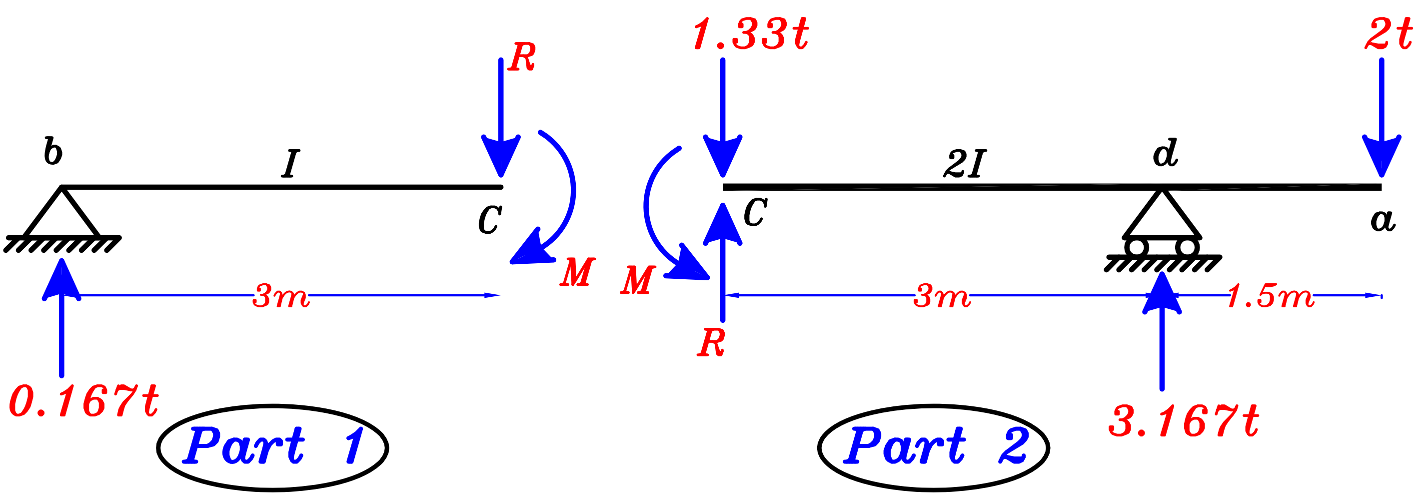
Example:

For the shown beam find the deflection at points (a&c) and the slope angle at points (c) . applying double integration ($EI = 20000 \ m^2.t.$)







و لحساب الـ $R\&M$ نطبق $\Sigma Y = 0$ و $\Sigma M = 0$ لاى من الجزئين و ليكن الجزء الاول .

$$\Sigma Y = 0 \Longrightarrow R = 0.167t$$

$$\Sigma M @ C = 0 \Longrightarrow M = -0.5m.t$$

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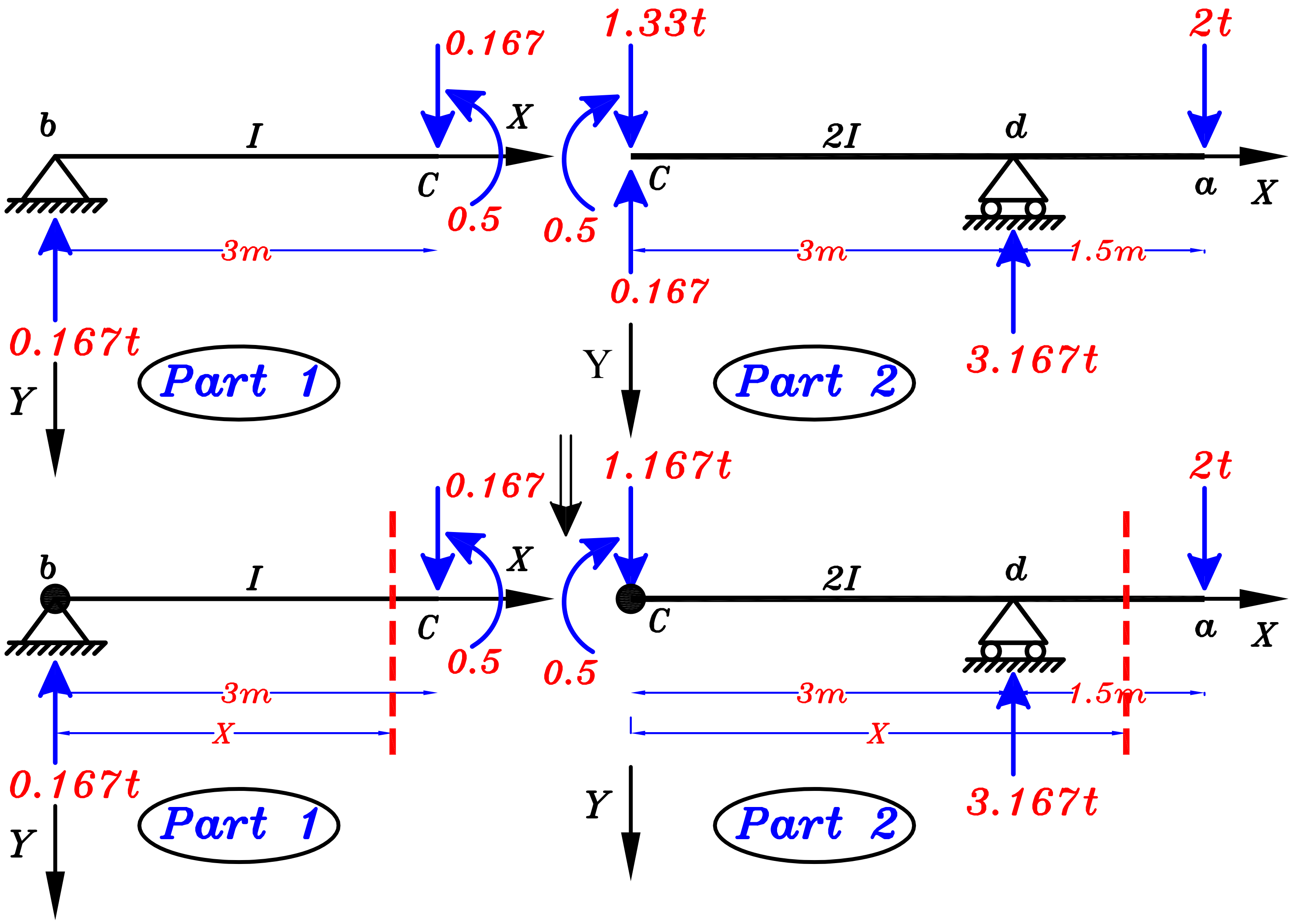

For part 1

$$M(x) = 0.167\ (x)$$

$$EIy'' = -M(x) \qquad EIy'' = -\ 0.167\ (x)$$

$$EIy' = -\ (0.167/2)\ (x)^2 + C_1$$

$$EIy = -\ (0.167/6)\ (x)^3 + C_1(x) + C_2$$

For part 2

$$M(x) = -\ 1.167\ (x) + 0.5\ (x)^0 + 3.167(x-3)$$

$$2EIy'' = -M(x) = 1.167\ (x) - 0.5\ (x)^0 - 3.167(x-3)$$

$$EIy'' = 0.574\ (x) - 0.25\ (x)^0 - 1.584(x-3)$$

$$EIy' = 0.287\ (x)^2 - 0.25\ (x)^1 - 0.792(x-3)^2 + C_3$$

$$EIy = 0.096\ (x)^3 - 0.125\ (x)^2 - 0.264(x-3)^3 + C_3 x + C_4$$

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To get C1 , C2 , C3 ,and C4

For part 1

at $X = 0\ m \Longrightarrow Y=0$ نعوض فى معادلة ال deflection ل Part 1

$$EI\,y = 0 = -(0.167/6)\,(0)^3 + C_1(0) + C_2 \Longrightarrow \boxed{C_2 = 0}$$

For part 2

at $X = 3\ m \Longrightarrow Y=0$ نعوض فى معادلة ال deflection ل Part 2

$$EI\,y = 0 = 0.096\,(3)^3 - 0.125\,(3)^2 - 0.264(3-3)^3 + C_3 3 + C_4$$

$$\boxed{3C_3 + C_4 = -1.497 \Longrightarrow Eq.(1)}$$

Y_c from part 1 $= Y_c$ from part 2

Y_c from part 1 (@x=3)

$$= \frac{1}{EI}\left\{-(0.167/6)\,(3)^3 + C_1(3) + C_2\right\}$$

$$= \frac{1}{EI}\left\{-(0.167/6)\,(3)^3 + C_1(3) + 0\right\}$$

$$= \frac{1}{EI}\left\{3C_1 - 0.752\right\}$$

Y_c from part 2 (@x=0)

$$= \frac{1}{EI}\left\{0.096\,(0)^3 - 0.125\,(0)^2 - 0.264\overset{-ve}{\cancel{(0-3)}}^3 + C_3 0 + C_4\right\}$$

$$= \frac{1}{EI}\left\{C_4\right\}$$

Y_c from part 1 $= Y_c$ from part 2

$$\frac{1}{EI}\left\{3C_1 - 0.752\right\} = \frac{1}{EI}\left\{C_4\right\}$$

$$\boxed{3C_1 - C_4 = 0.752 \Longrightarrow Eq.(2)}$$

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Y'_C from part 1 $= Y'_C$ from part 2

Y'_C from part 1 (@x=3)

$$= \frac{1}{EI}\left\{-(0.167/2)(3)^2 + C_1\right\} = \frac{1}{EI}\left\{-0.752 + C_1\right\}$$

Y'_C from part 2 (@x=0)

$$= \frac{1}{EI}\left\{0.287\,(0)^2 - 0.25\,(0)^1 - 0.792\cancel{(0-3)}^{\,-ve\;2} + C_3\right\}$$

$$= \frac{1}{EI}\left\{C_3\right\}$$

Y'_C from part 1 $= Y'_C$ from part 2

$$\frac{1}{EI}\left\{-0.752 + C_1\right\} = \frac{1}{EI}\left\{C_3\right\}$$

$$\boxed{C_1 - C_3 = 0.752 \Longrightarrow \text{Eq.(3)}}$$

Solving equations 1,2,3

$\boxed{C_1 = 0.25}$ $\boxed{C_3 = -0.50}$ $\boxed{C_4 = 0.0035}$

For part 1

$$EIy' = -(0.167/2)(x)^2 + 0.25$$

$$EIy = -(0.167/6)(x)^3 + 0.25\,(x)$$

For part 2

$$EIy' = 0.287\,(x)^2 - 0.25\,(x)^1 - 0.792(x-3)^2 - 0.50$$

$$EIy = 0.096\,(x)^3 - 0.125\,(x)^2 - 0.264(x-3)^3 - 0.5x + 0.0035$$

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At point (C)

لحساب ال *deflection* نعوض فى معادلة ال *deflection*
و لان نقطة *C* تقع على الحد الفاصل بين *Part 1&2* لذا نعوض فى معادلة
ال *deflection* لاى من الجزئين و ليكن *Part 1* .

at Point C $\Longrightarrow$ $X = 3\ m$

$$EIy = -(0.167/6)\ (3)^3 + 0.25\ (3) = -\ 0.0035$$

$$y_c = \frac{-0.0035}{EI} = \frac{-0.0035}{20000} = \textbf{Zero}$$ لان الرقم صغير جدا

لحساب ال *Slope angle* نعوض فى معادلة ال *Slope angle*
و لان نقطة *C* تقع على الحد الفاصل بين *Part 1&2* لذا نعوض فى معادلة
ال *deflection* لاى من الجزئين و ليكن *Part 1* .

$$EIy' = -(0.167/2)\ (3)^2 + 0.25 = -0.502$$

$$y'_c = \frac{-0.502}{EI} = \frac{-0.502}{20000} = \mathbf{-0.000025\ rad.}$$

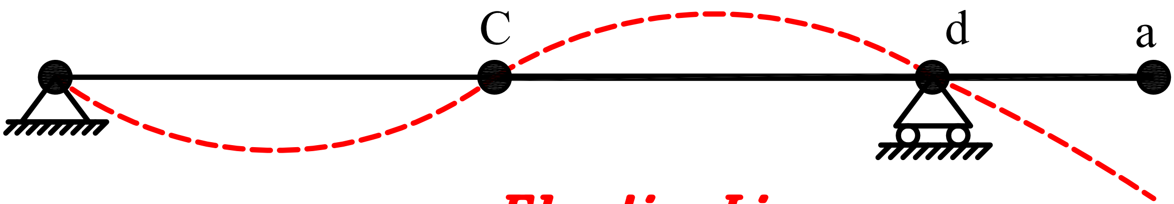
At point (a)

لحساب ال *deflection* نعوض فى معادلة ال *deflection*
و لان نقطة *C* تقع فى *Part 2* لذا نعوض فى معادلة *Part 2*

at Point a $\Longrightarrow$ $X = 4.5\ m$

$$EIy = 0.096\ (4.5)^3 - 0.125\ (4.5)^2 - 0.264(4.5-3)^3$$
$$-0.5x4.5 + 0.0035 = 7.58$$

$$y_a = \frac{3.10}{EI} = \frac{3.10}{20000} = \mathbf{0.000155\ m = 0.0155\ cm}$$



Elastic Line

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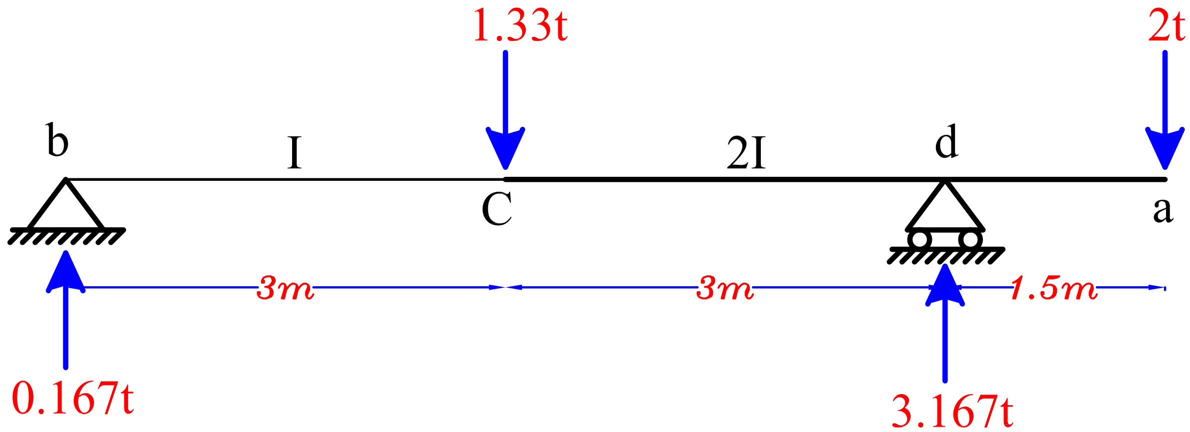
طريقة أخرى للحل

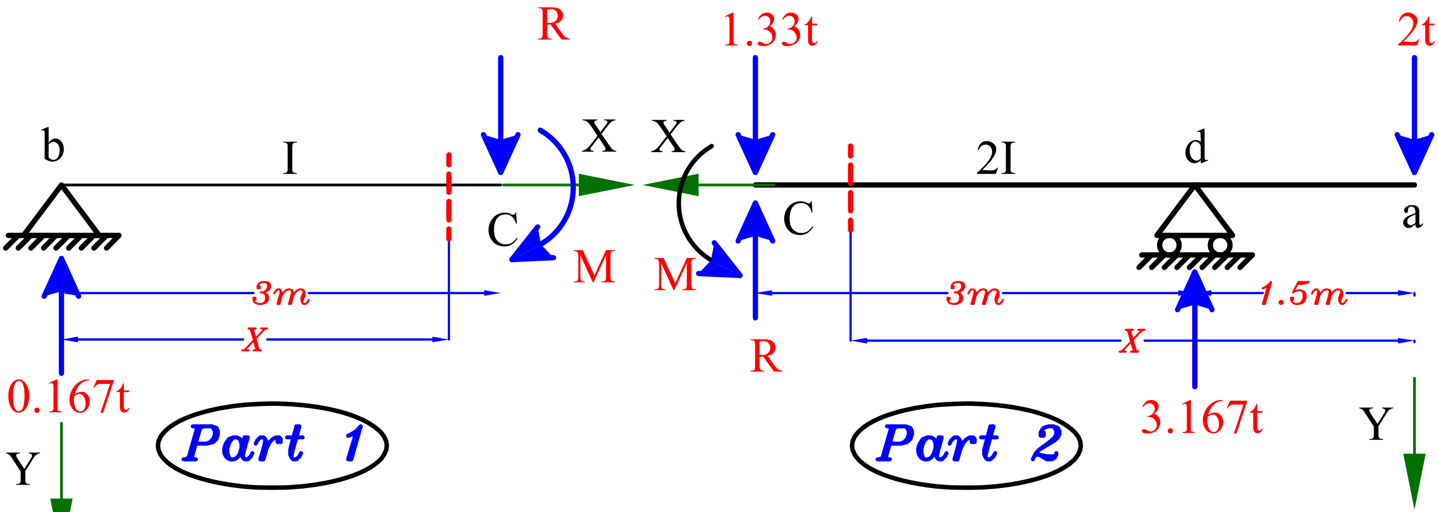
و هى أن نفرض فى الجزء الاول محور X لليمين و فى الجزء الثانى نفرض محور X لليسار .

و لكن فى هذه الحالة عند مساواة الـ Slope angle عند نقطة a من اليمين و اليسار تكون الاشارة معكوسة و ذلك لان المحاور معكوسة و لكن فى الـ deflection لا نعكس الاشارات .

$$Y'_c \text{ from part } 1 = - Y'_c \text{ from part } 2$$

$$Y_c \text{ from part } 1 = Y_c \text{ from part } 2$$





For part 1 :

M (x) = 0.166 (x)

EI y'' = - M (x)   EI y'' = - 0.166 (x)

EI y' = - (0.166 / 2) (x)² + C1

EI y = - (0.166 / 6) (x)³ + C1 (x) + C2

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From intial conditions:

At x = 0 --------- y = 0 ----------------- $\boxed{C2 = 0}$

At x = 3 m ------ y = y @ c & y' = y' @ c

y @ c = (-0.75 + 3 C1) / EI ---------------------- (1)

y ' @ c = (-0.75 + C1) / EI ---------------------- (2)

For part 2 :

M (x) = 3.166 (x - 1.5) - 2 (x)

2EI y'' = - M (x)

2EI y'' = - 3.166 (x - 1.5) + 2 (x)

2EI y' = - (3.166 / 2) (x - 1.5)^2 + (x)^2 + C3

2EI y = - (3.166 / 6) (x - 1.5)^3 + (x)^3 / 3 + C3 (x) + C4

From intial conditions:

At x = 1.5 m -------------- y = 0

0 = 1.125 + 1.5 C3 + C4 ------------ C4 = - (1.125 + 1.5 C3) ----- (3)

At x = 4.5 m -------------- y = y @ c & y' = y' @ c

y @ c = (16.125 + 4.5 C3 + C4) / 2EI

Y ' @ c = (16.125 + 4.5 C3 - 1.125 - 1.5 C3) /2EI

= (15 + 3 C3) / 2EI --------------- (4)

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y' @ c = (6 + C3) / 2EI ----------------------(5)

y @ c (right) = y @ c (left)

y' @ c (right) = - y' @ c (left) لان المحاور معكوسة

C1 = 0.25	C3 = -5	C4 = 6.375

For part 1 :

EI y' = -0.166 (x)² / 2 + 0.25

EI y = -0.166 (x)³ / 6 + 0.25 (x)

y @ c = 0

y' @ c = (- 0.5 / EI) = -0.000025 rad.

For part 2 :

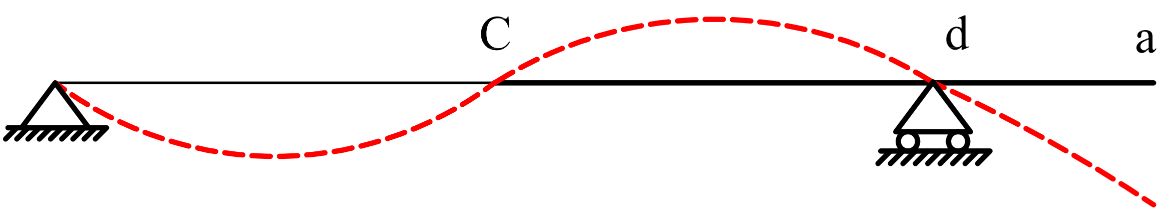
2EI y' = (x)² - 1.5833 (x - 1.5)² - 5

2EI y = (x)³ / 3 - 0.527766 (x - 1.5)³ - 5 x + 6.375

At point (a) : x = 0 ----------------- y = 3.1875 / EI = 0.00016 m

At point (d) : x = 1.5 ----------------- y' = -1.375 / EI = 1.375 / EI

لان المحاور معكوسة



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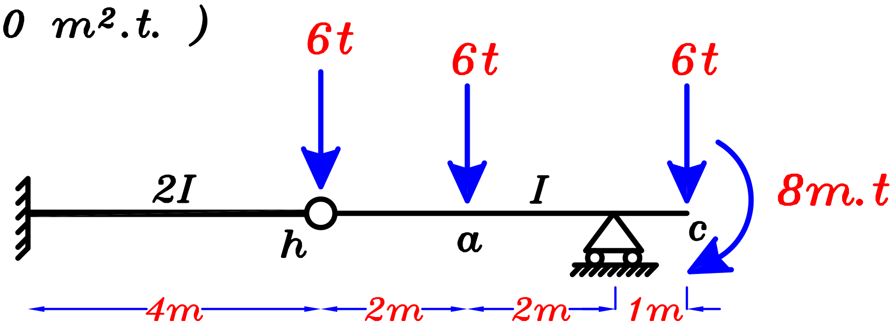
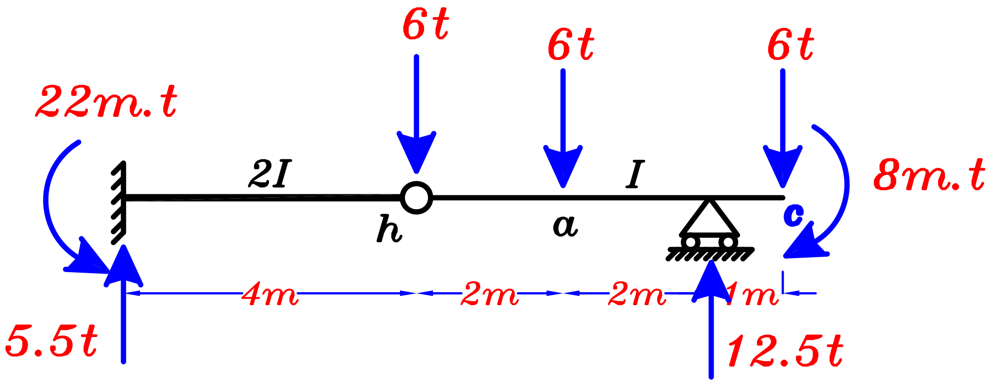
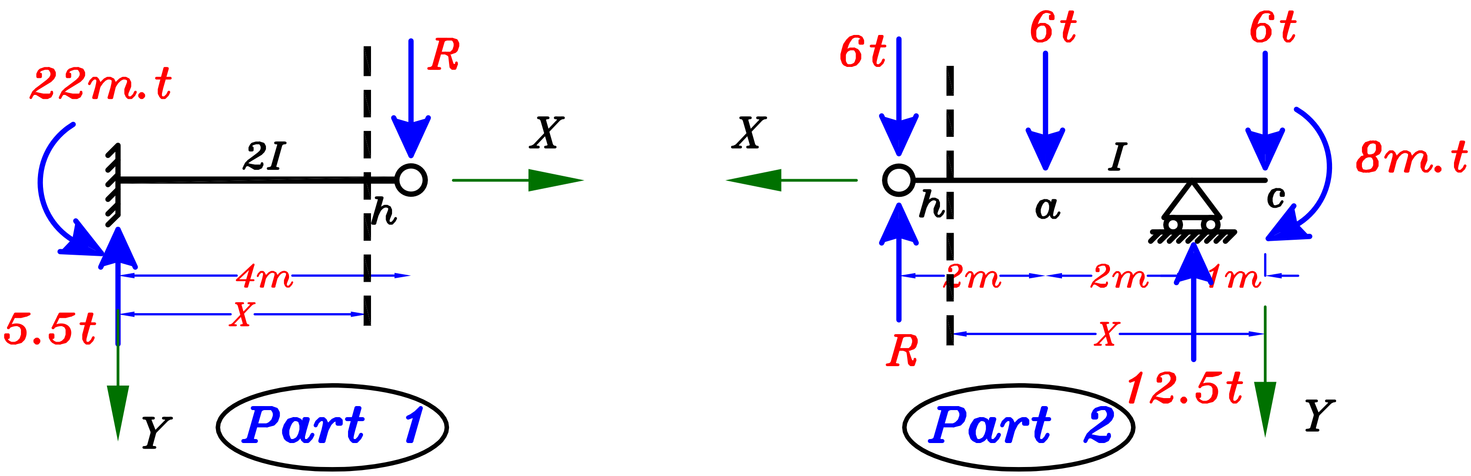
Example:

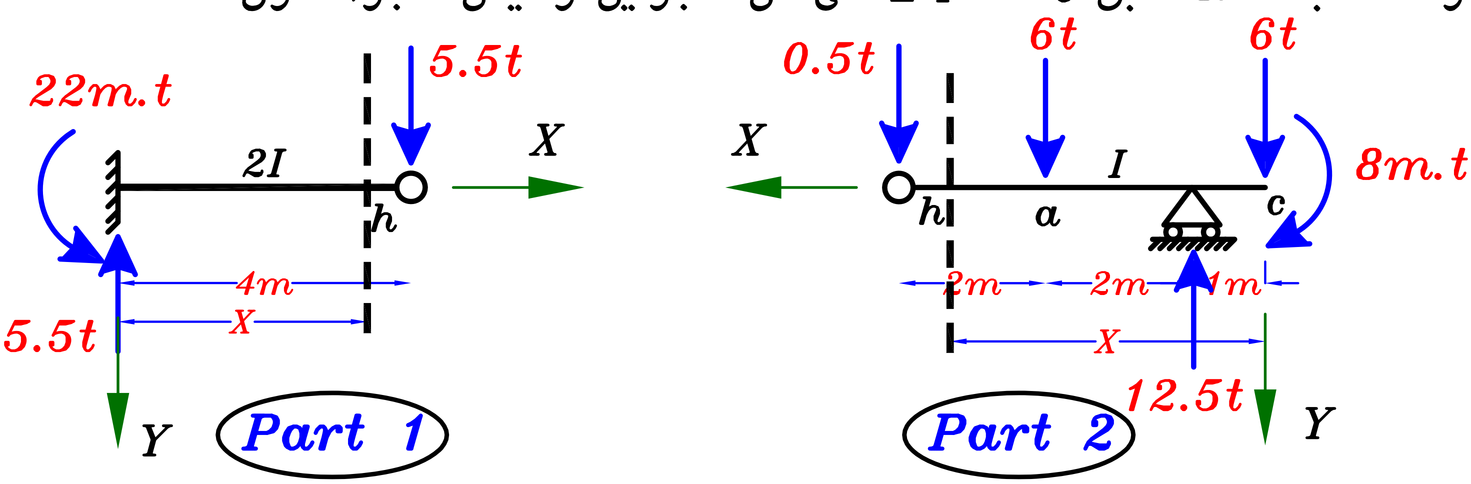
For the shown beam find the deflection at points (a&c) and the slope angle at points (c) and the change in slope at point (h) . applying double integration method .
($EI = 20000\ m^2.t.$)







و لحساب ال R نطبق $\Sigma Y = 0$ لای من الجزئين و ليكن الجزء الاول .



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For part 1

$$M(x) = 5.5\ (x) - 22\ (x)^0$$

$$EIy'' = -M(x)$$

$$2EIy'' = -5.5\ (x) + 22\ (x)^0$$

$$EIy'' = -2.75\ (x) + 11\ (x)^0$$

$$EIy' = -1.375\ (x)^2 + 11\ (x)^1 + C_1$$

$$EIy = -0.4583\ (x)^3 + 5.5\ (x)^2 + C_1 x + C_2$$

From intial conditions:

at $X = 0\ m \Longrightarrow Y=0 \Longrightarrow \boxed{C_2 = 0}$

at $X = 0\ m \Longrightarrow Y' = 0 \Longrightarrow \boxed{C_1 = 0}$

$$EIy' = -1.375\ (x)^2 + 11\ (x)^1$$

$$EIy = -0.4583\ (x)^3 + 5.5\ (x)^2$$

At point (h)

at Point $h \Longrightarrow X = 4\ m$

$$y_C = \frac{58.66}{EI} = \frac{58.66}{20000} = 0.0029\ m = 0.29\ cm$$

$$y'_{C\ left} = \frac{22}{EI} = \frac{22}{20000} = 0.0011\ rad.$$

For part 2

$$M(x) = 12.5(x-1) - 6(x) - 6(x-3) - 8\ (x)^0$$

$$EIy'' = -M(x)$$

$$EIy'' = -12.5(x-1) + 6(x) + 6(x-3) - 8\ (x)^0$$

$$EIy' = -6.25(x-1)^2 + 3(x)^2 + 3(x-3)^2 - 8\ (x) + C_3$$

$$EIy = -2.083(x-1)^3 + (x)^3 + (x-3)^3 - 4\ (x)^2 + C_3 x + C_4$$

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From intial conditions

at $X = 0\ m \Longrightarrow Y=0 \Longrightarrow C_3 + C_4 = -5 \Longrightarrow$ Eq.(1)

Y_h from part 1 $= Y_h$ from part 2

Y_h from part 1 (@x=4)

$$= \frac{1}{EI}\left\{-\ 0.4583\ (4)^3 + 5.5\ (4)^2 + C_1 4 + C_2\right\}$$

$$= \frac{1}{EI}\left\{4C_1 + C_2 + 58.67\right\} = \frac{1}{EI}\left\{0 + 0 + 58.67\right\}$$

$$= \frac{1}{EI}\left\{58.67\right\}$$

Y_h from part 2 (@x=5)

$$= \frac{1}{EI}\left\{-\ 2.083(5-1)^3 + (5)^3 + (5-3)^3 - 4\ (5)^2 + C_3 5 + C_4\right\}$$

$$= \frac{1}{EI}\left\{5C_3 + C_4 - 99.66\right\}$$

Y_h from part 1 $= Y_h$ from part 2

$$\frac{1}{EI}\left\{58.67\right\} = \frac{1}{EI}\left\{5C_3 + C_4 - 99.66\right\}$$

$5C_3 + C_4 = 158.33 \Longrightarrow$ Eq.(2)

By solving the two equations

$\boxed{C_3 = 40.83}$ $\boxed{C_3 = -45.83}$

For part 2

$$EIy' = -\ 6.25(x-1)^2 + 3(x)^2 + 3(x-3)^2 - 8\ (x) + 40.83$$

$$EIy = -\ 2.083(x-1)^3 + (x)^3 + (x-3)^3 - 4\ (x)^2 + 40.83\ x - 45.83$$

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At point (a)

at Point a $\Longrightarrow$ $X = 3\ m$

$$y_a = \frac{50.99}{EI} = \frac{50.99}{20000} = \mathbf{0.0025\ m} = \mathbf{0.25\ cm}$$

At point (C)

at Point C $\Longrightarrow$ $X = 0\ m$

$$y_C = \frac{-45.83}{EI} = \frac{-45.83}{20000} = \mathbf{0.0023\ m} = \mathbf{0.23\ cm}$$

$$y'_C = \frac{40.83}{EI} = \frac{40.83}{20000} = \mathbf{0.002\ rad.}$$

$$y'_C = -\ 0.002\ rad.$$

و لكن لان المحاور معكوسة

At point (h)

at Point h $\Longrightarrow$ $X = 5\ m$

$$y_h = \frac{58.67}{EI} = \frac{58.67}{20000} = 0.0029\ m = \mathbf{0.29\ cm}$$

$$y'_{h\,right} = \frac{-92.17}{EI} = \frac{-92.17}{20000} = \mathbf{-0.0046\ rad.}$$

$$y'_{h\,right} = \mathbf{0.0046\ rad.}$$

و لكن لان المحاور معكوسة

Change in slope angle at (h)

$$= y'_{h\,right} - y'_{h\,left} = 0.0046\ rad. - 0.0011\ rad.$$

$$= \mathbf{0.0035\ rad.}$$

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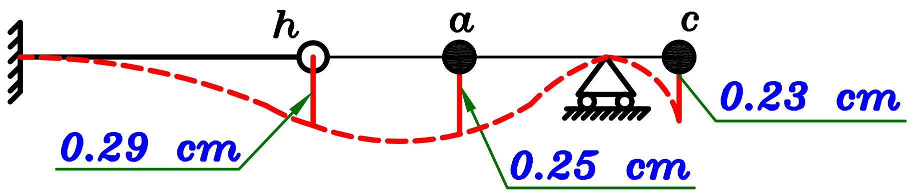

Elastic Line

خد بالك

فى هذه المسألة لا يمكن تطبيق Y'_h *from part 1* $= Y'_h$ *from part 2*
و ذلك لان الـ *Slope angle* يمين و شمال نقطة h غير متساوى و لذلك نحسب
الـ *Change in slope angle at (h)*.

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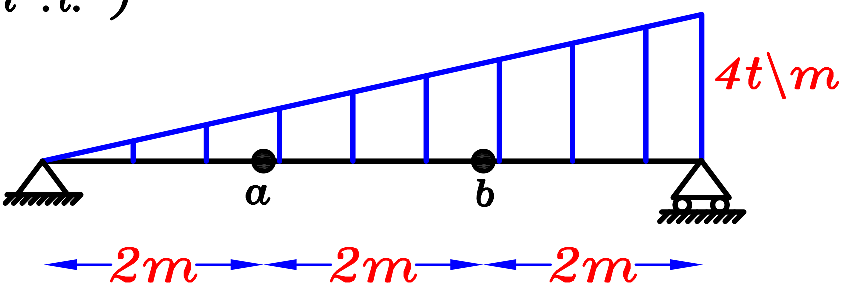
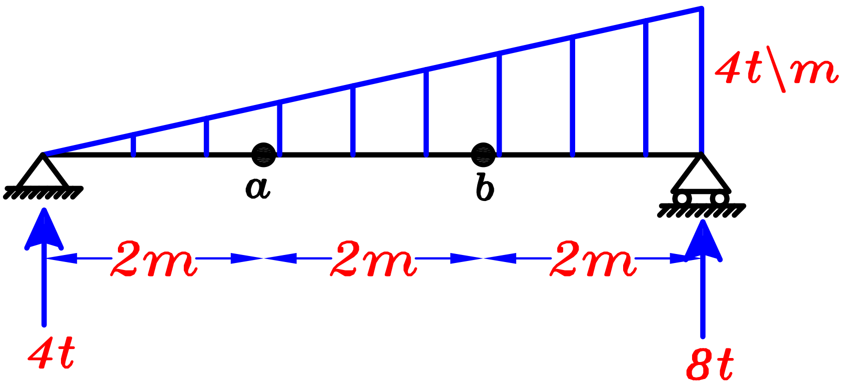
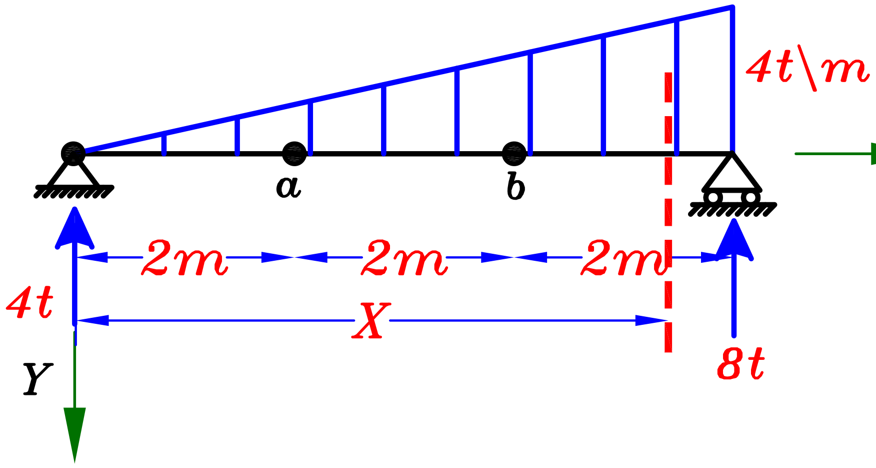
Example:

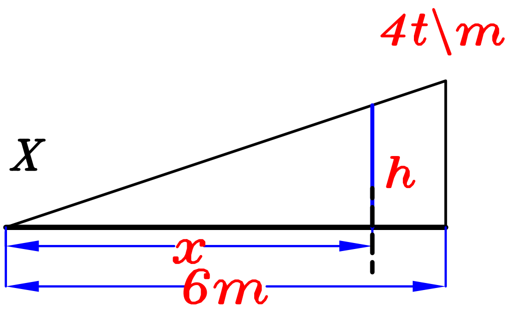
For the shown beam find the deflection at points (a&c) and

For the shown beam find the deflection at points (a&b) .
applying the method of double integration.
($EI = 20000\ m^2.t.$)









$\boxed{h/x = 4/6}$

$$M(x) = 4(x) - (4/6)\ (x)\ (x/2)\ (x/3)$$

$$= 4(x) - (1/9)\ (x)^3$$

$$EIy'' = -M(x)$$

$$EIy'' = -4(x) + (1/9)\ (x)^3$$

$$EIy' = -2(x)^2 + (1/36)\ (x)^4 + C_1$$

$$EIy = -0.67(x)^3 + (1/180)^5 (x) + C_1 x + C_2$$

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From intial conditions:

at $X = 0$ m $\Longrightarrow$ $Y=0$ $\Longrightarrow$ $\boxed{C_2 = 0}$

at $X = 6$ m $\Longrightarrow$ $Y=0$ $\Longrightarrow$ $\boxed{C_1 = 16.8}$

$$EIy' = -2(x)^2 + (1/36)(x)^4 + 16.8$$

$$EIy = -0.67(x)^3 + (1/180)^5(x) + 16.8\,x$$

At point (a)

at Point a $\Longrightarrow$ $X = 2$ m

$$y_a = \frac{29.68}{EI} = \frac{29.68}{20000} = 0.00148 \text{ m} = 0.148 \text{ cm}$$

At point (b)

at Point b $\Longrightarrow$ $X = 4$ m

$$y_a = \frac{30.22}{EI} = \frac{30.22}{20000} = 0.00151 \text{ m} = 0.151 \text{ cm}$$

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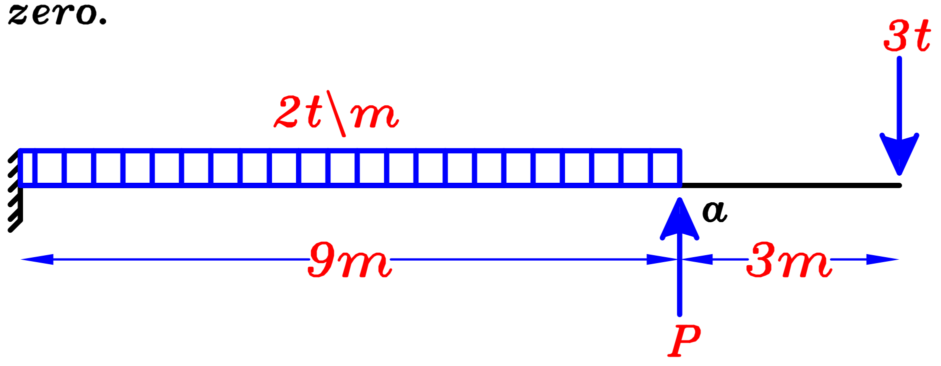
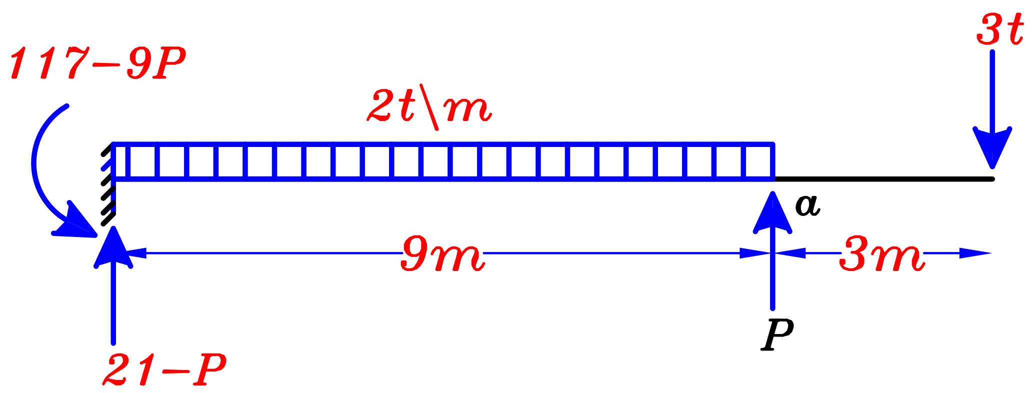
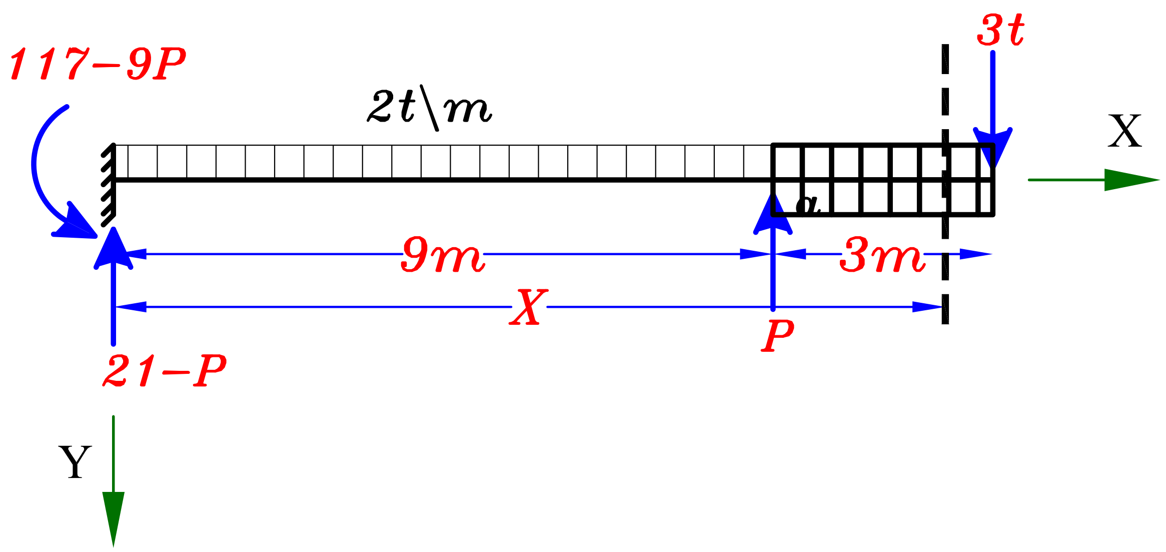
Example:

For the shown beam and applying maculli's method , find the value of (p) so that the deflection at point (a) equals zero.







$$M(x) = (21-P)\ (x) - 2(x)\ (x/2) - (117-9P) + P\ (x-9)$$
$$+2\ (x-9)\ (x-9)\ /\ 2$$

$$M(x) = (12-P)\ (x) - (x)^2 - (117-9P)\ (x)^0 + P\ (x-9)$$
$$+\ (x-9)^2$$

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$$EIy'' = -M(x)$$

$$EIy'' = -(21-P)\ (x) + (x)^2 + (117-9P)\ (x)^0 - P(x-9) - (x-9)^2$$

$$EIy' = -(21-P)\ (x)^2/\ 2 + (x)^3/\ 3 + (117-9P)\ (x)^1 - (P/2)\ (x-9)^2 - (x-9)^3/\ 3 + C_1$$

$$EIy = -(21-P)\ (x)^3/\ 6 + (x)^4/\ 12 + (117-9P)\ (x)^2/\ 2 - (P/6)\ (x-9)^3 - (x-9)^4/\ 12 + C_1(x) + C_2$$

From intial conditions:

at $X = 0\ m \Longrightarrow Y=0 \Longrightarrow \boxed{C_2 = 0}$

at $X = 0\ m \Longrightarrow Y' = 0 \Longrightarrow \boxed{C_1 = 0}$

$$EIy' = -(21-P)\ (x)^2/\ 2 + (x)\ /\ 3 + (117-9P)\ (x)^1 - (P/2)\ (x-9)^2 - (x-9)^3/\ 3$$

$$EIy = -(21-P)\ (x)^3/\ 6 + (x)^4/\ 12 + (117-9P)\ (x)^2/\ 2 - (P/6)\ (x-9)^3 - (x-9)^4/\ 12$$

At point (a)

at Point a $\Longrightarrow X = 9\ m \Longrightarrow Y = 0$

لان الـ Support عند نقطة a يمنع الحركة الرأسية و بالتالى فان الـ deflection يساوى صفر .

$$EIy = 0 = -(21-P)\ (9)^3/\ 6 + (9)^4/\ 12 + (117-9P)\ (9)^2/\ 2 - (P/6)\ (9-9)^3 - (9-9)^4/\ 12$$

$$\boxed{P = 11.25\ t}$$

و هذا معناه أنه يمكننا استخدام معادلات الـ deflection فى حل المنشات الـ Indeterminate لان هذا المنشأ Indeterminate و هذا سوف نتعلمه فيما بعد

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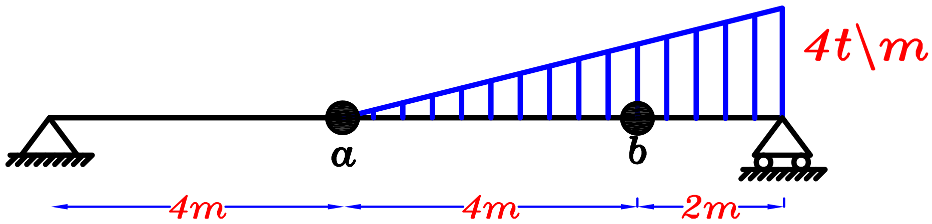
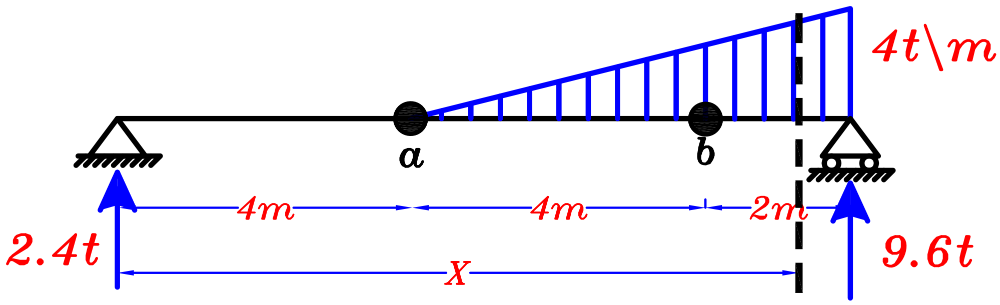
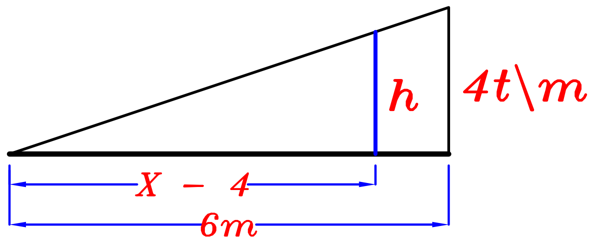
Example:

For the shown beam find the deflection at points (a&b) . applying the method of double integration.





$$\boxed{\frac{X-4}{h} = \frac{6}{4}}$$



$$M(x) = 2.4(x) - (4/6)\ (x-4)\ [(x-4)/2]\ [(x-4)/3]$$

$$M(x) = 2.4(x) - (1/9)\ (x-4)^3$$

$$EIy'' = -M(x)$$

$$EIy'' = -2.4\ (x) + (1/9)\ (x-4)^3$$

$$EIy' = -1.2\ (x)^2 + (1/36)\ (x-4)^4 + C_1$$

$$EIy' = -0.4\ (x)^3 + (1/180)\ (x-4)^5 + C_1 x + C_2$$

From intial conditions:

at $X = 0$ m $\Longrightarrow$ $Y=0$ $\Longrightarrow$ $\boxed{C_2 = 0}$

at $X = 10$ m $\Longrightarrow$ $Y=0$ $\Longrightarrow$ $\boxed{C_1 = 35.68}$

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$$EIy' = -1.2\ (x)^2 + (1/36)\ (x-4)^4 + 35.68$$

$$EIy' = -0.4\ (x)^3 + (1/180)\ (x-4)^5 + 35.68\ x$$

At point (a)

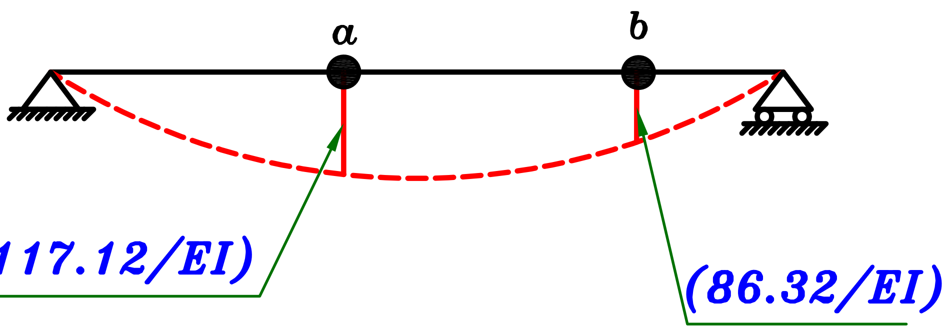
at Point a $\Longrightarrow$ $X = 4\ m$

$$y_a = \frac{117.12}{EI}$$

At point (b)

at Point b $\Longrightarrow$ $X = 8\ m$

$$y_b = \frac{86.32}{EI}$$



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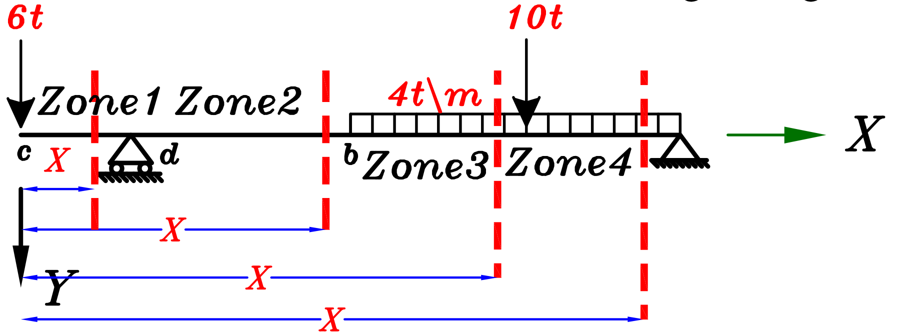
Method of Zones

توجد طريقة أخرى لحل المسائل تسمى بال *Method of Zones* و تقوم فكرتها على التالى :



نقسم المسألة الى *Zones* و ذلك عند كل *Support* أو *Load* أو *Intermediate hinge* أو تغير فى ال *Inertia* و نكتب معادلة لل *moment* فى نهاية كل *Zone* .

دائما العلاقة الرابطة بين كل *Zone* و الاخرى هى أن $Y_{left} = Y_{right}$ & $\grave{Y}_{left} = \grave{Y}_{right}$ عدا فى حالة ما اذا كان الفاصل *Intermediate hinge* تكون $Y_{left} = Y_{right}$.

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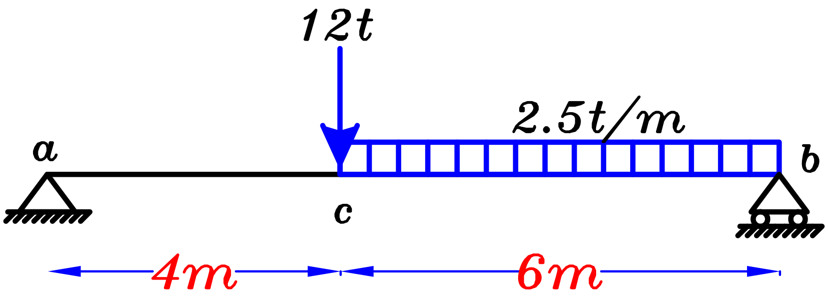
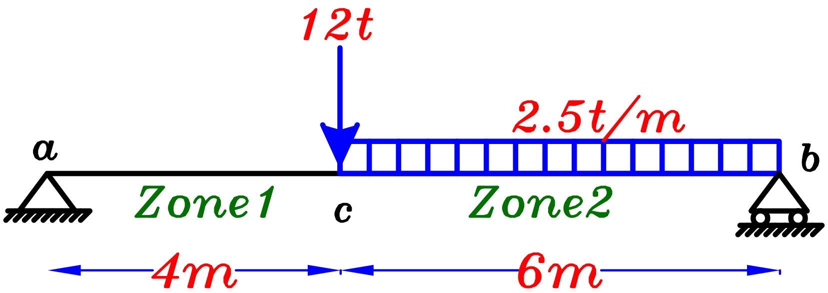
Example:

Using zones find the deflection at point c & slope angle at (a&b) , then find the maximum deflection using zones method.

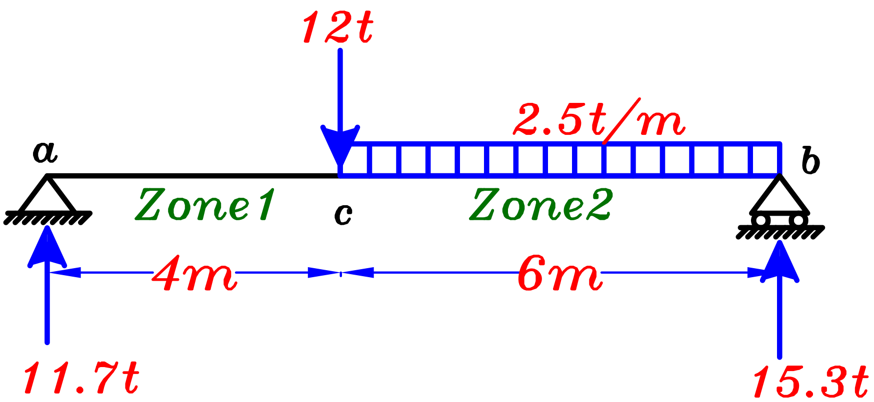


هذه طريقة مختلفة قليلا يتم فيها تقسيم المسألة الى Zones

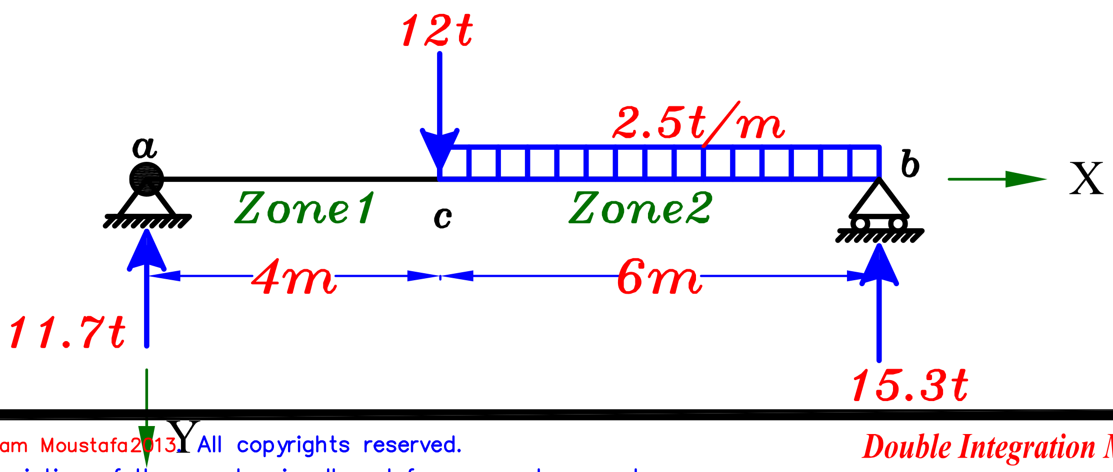
١- نقسم المسألة الى Zones و الحد الفاصل بين الـZones يكون Force أو Support أو Intermediate hinge أو تغير الـInertia.



٢- نحسب الـReactions.



٣- نفرض محور X لليمين و محور Y للاسفل.



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٤- نكتب معادلة ال Moment في Zone 1 على اساس أن ال Origin هو نقطة a .

$$M(1) = 11.7\ x$$

٥- نكتب معادلة ال Moment في Zone 2 على اساس أن ال Origin هو نقطة a .

$$M(2) = 11.7x - 12(x-4) - 2.5(x-4)^2 / 2$$

٦- نحسب في كل Zone ال Slope angle(Y`) و ال Deflection(Y)

و في هذه الطريقة يمكن فك الاقواس

$$EIy''_{(1)} = -11.7\ x$$

$$EIy'_{(1)} = -11.7\ x^2 / 2 + C_1$$

$$EIy_{(1)} = -11.7\ x^3 / 6 + C_1 x + C_2$$

$$EIy''_{(2)} = -11.7x + 12x - 48 + 1.25(x^2 - 8x + 16)$$

$$= 1.25x^2 - 9.7x - 28$$

$$EIy'_{(2)} = 1.25(x^3 / 3) - 9.7\ (x^2 / 2) - 28\ x + C_3$$

$$EIy_{(2)} = 1.25(x^4/12) - 9.7(x^3 / 6) - 14x^2 + C_3 x + C_4$$

$$= 0.104167x^4 - 1.61617x^3 - 14x^2 + C_3 x + C_4$$

٧- لايجاد ال Boundary Conditions

يتم التعويض بكل نقطة في ال Zone الخاصة بها

From intial conditions

at $X = 0\ m \Longrightarrow Y=0 \Longrightarrow \boxed{C_2 = 0} \Longrightarrow$ Zone 1

at $X = 10m \Longrightarrow Y=0 \Longrightarrow$ Zone 2

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$$0 = 0.104167(10)^4 - 1.6167(10)^3 - 14(10)^2 + C_1(10) + C_2 \longrightarrow Eq.(1)$$

---

at $X = 4\ m \Longrightarrow y_{C\,Left} = y_{C\,Right}$

لان ال deflection عند نقطة C فى Zone1 يساوى Zone2 .

$$-1.95(4)^3 + C_1(4) = 0.104167(4)^4 - 1.6167(4)^3 - 14(4)^2 + C_3(4) + C_4 \longrightarrow Eq.(2)$$

---

at $X = 4\ m \Longrightarrow y'_{C\,Left} = y'_{C\,Right}$

لان ال Slope angle عند نقطة C فى Zone1 يساوى Zone2 .

$$-5.85(4)^2 + C_1 = 0.4167(4)^3 - 4.85(4)^2 - 28(4) + C_3 \longrightarrow Eq.(3)$$

Solving the equations:

C1 = 138.3 & C3 = 207.633 & C4 = -101.33

$\boxed{C_1 = 138.3}$ $\boxed{C_3 = 207.633}$ $\boxed{C_4 = -101.33}$

For Zone (1)

$$EIy'_{(1)} = -11.7\ x^2/\ 2 + 138.3$$

$$EIy_{(1)} = -11.7\ x^3/\ 6 + 138.3\ x$$

For Zone (2)

$$EIy'_{(2)} = 1.25(x^3/\ 3) - 9.7\ (x^2/\ 2) - 28\ x + 207.63$$

$$EIy_{(2)} = 1.25(x^4/12) - 9.7(x^3/\ 6) - 14x^2 + 207.63x - 101.33$$

٨- يتم التعويض بكل نقطة فى ال Zone الخاصة بها

أى نقطة From(X=0 to X=4) يتم التعويض فى معادلات Zone1

أى نقطة From(X=0 to X=10) يتم التعويض فى معادلات Zone2

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For Zone (1)

At point (a)

at Point a $\Longrightarrow$ $X = 0\ m$

$$y'_a = \frac{183.3}{EI}$$

At point (C)

at Point C $\Longrightarrow$ $X = 4\ m$

$$y_c = \frac{428.4}{EI} \qquad y'_c = \frac{44.7}{EI}$$

For Zone (2)

At point (b)

at Point b $\Longrightarrow$ $X = 10\ m$

$$y'_c = \frac{-101.27}{EI}$$

At point (C)

at Point C $\Longrightarrow$ $X = 4\ m$

$$y_c = \frac{428.4}{EI} \qquad y'_c = \frac{44.7}{EI}$$

و للحصول على ال *maximun deflection* نفاضل معادلة ال *deflection* و نساويها بالصفر فى كل *Zone* .

Try $Y'(1) = 0$

$-5.85\ x^2 + 138.3 = 0 \Longrightarrow x = +\ 4.86 \quad \& \quad -4.86$

الجواب الموجب مرفوض لان (x) تقع خارج *(Zone 1)*

الجواب السالب مرفوض لنفس السبب

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Try $Y(2)^{\backprime} = 0$

$$0.4167x^3 - 4.85x^2 - 28x + 207.63 = 0$$

و لحل المعادلة التكعيبية نقوم بالاتى (طريقة الدكتور)

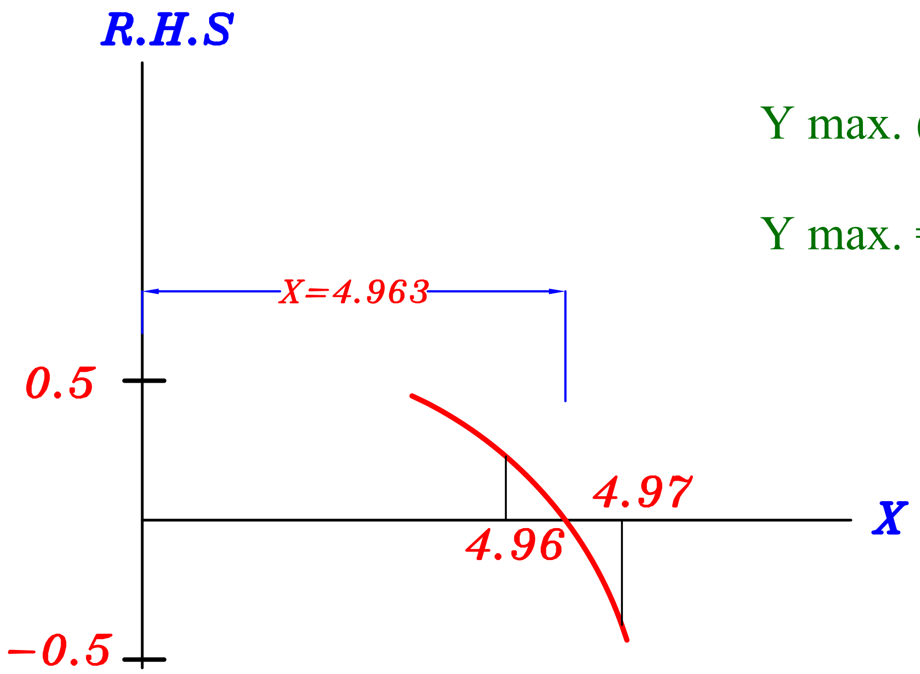
$$put\ 0.4167x^3 - 4.85x^2 - 28x + 207.633 = R.H.S$$

و نعوض بقيم لل (X) حتى نحصل على ال $R.H.S$ يساوى صفر

و من الممكن أن نعوض بأوبع أو خمس نقاط ثم نرسم علاقة بين ال (X)

و ال $R.H.S$ و تقاطع ال*Curve* مع الخط الافقى نحصل على قيمة ال (X)

X	5	5.1	4.9	4.96	4.97
R.H.S	−1.523	−6.04	3.01	0.28	−.58



Y max. @ x = 4.963

Y max. = 449.8 / EI

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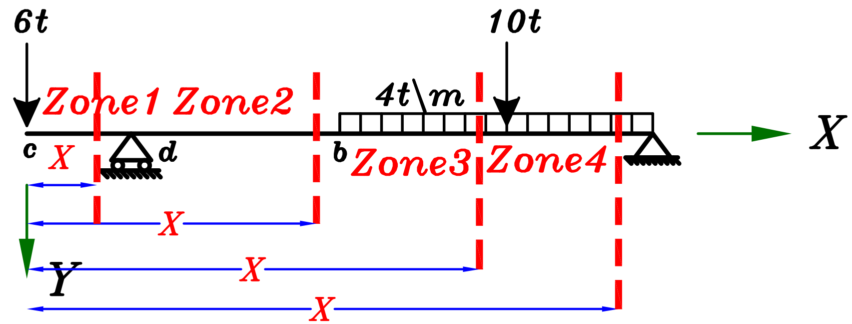
ملاحظات هامة جدا

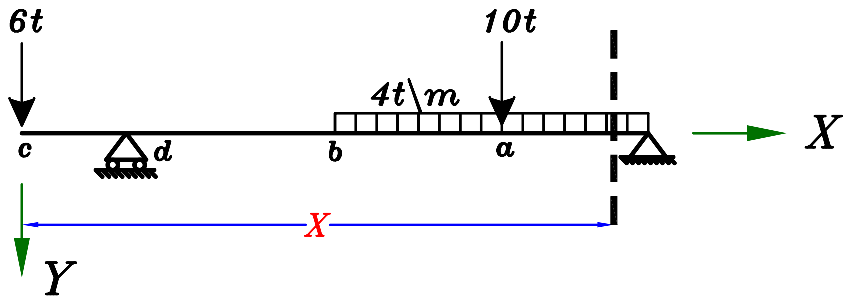
Method of Zones



نقسم المسألة الى *Zones* و ذلك عند كل *Support* أو *Load* أو *Intermediate hinge* أو تغير فى ال *Inertia* و نكتب معادلة للـ *moment* فى نهاية كل *Zone* .

دائما العلاقة الرابطة بين كل *Zone* و الاخرى هى أن $Y_{left} = Y_{right}$ & $Y'_{left} = Y'_{right}$ عدا فى حالة ما اذا كان الفاصل *Intermediate hinge* تكون $Y_{left} = Y_{right}$

Double integration method



نأخذ الـ *Section* قبل نهاية الكمرة مباشرة و نكتب معادلة واحدة للـ *moment* عند هذا الـ *Section* .

لا نفسم المسألة الا فى حالة وجود تغير فى الـ*Inertia* أو وجود *Intermediate hinge*

فى حالة وجود تغير فى الـ *Inertia* أو وجود *Intermediate hinge* نفصل الكمرة الى أكثر من جزء عند هذه الاماكن و تكون العلاقة الرابطة بين هذه الاجزاء أن الـ $Y_{left} = Y_{right}$ & $Y'_{left} = Y'_{right}$ فى حالة تغير الـ *Inertia* و أن الـ $Y_{left} = Y_{right}$ فى حالة وجود *Intermediate hinge* .

و سوف نحل المسألة القادمة بالطريقتين لتحديد الفرق بينهما

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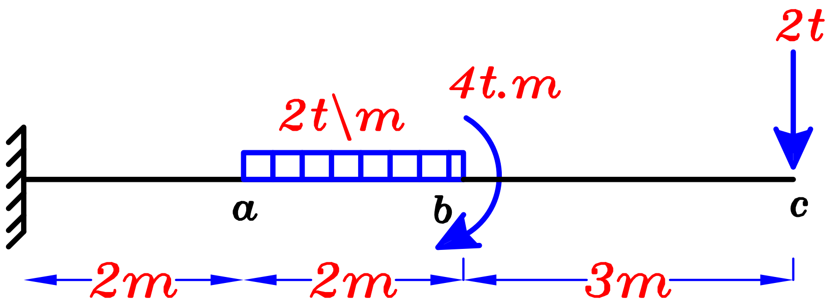
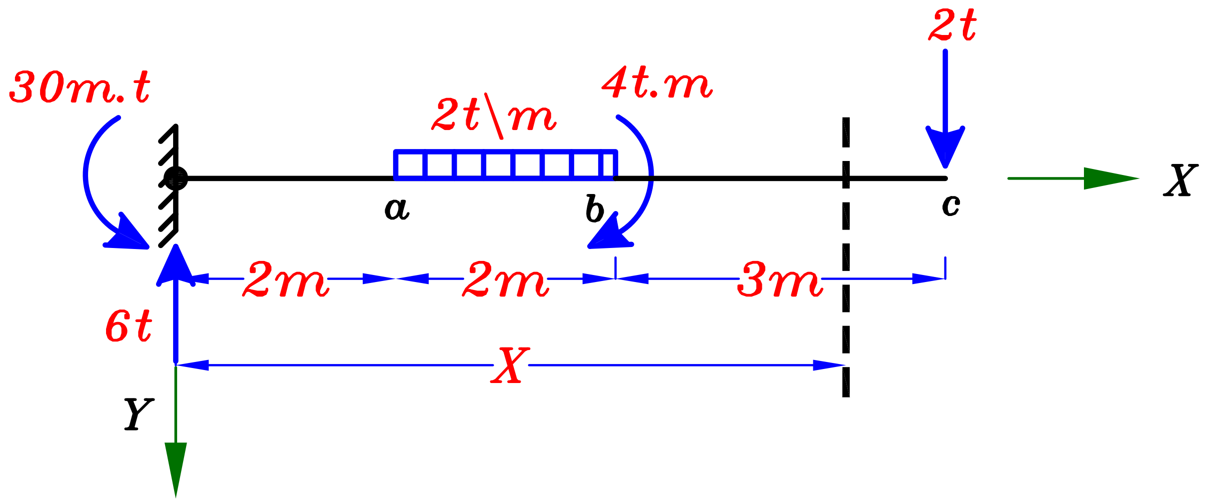
Example:

For the shown beam find the deflection at points (a&b&c) and the slope angle at points (c).applying double integration method. (EI = 20000 m^2.t.)





$$M(x) = 6(x) - 2(x-2)\,(x-2)/2 + 4(x-4)^{0} - 30(x)^{0} + 2\,(x-4)\,(x-4)\,/\,2$$

$$EIy'' = -M(x)$$

$$EIy'' = -6\,(x) + 2(x-2)\,(x-2)/2 - 4(x-4)^{0} + 30(x)^{0} - 2\,(x-4)\,(x-4)\,/\,2$$

$$EIy' = -3\,(x)^{2} + (x-2)^{3}/\,3 - 4(x-4) + 30(x) - (x-4)^{3}/\,3 + C_1$$

$$EIy = -(x)^{3} + (x-2)^{4}/\,12 - 2(x-4)^{2} + 15(x)^{2} - (x-4)^{4}/\,12 + C_1 x + C_2$$

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From intial conditions

at $X = 0\ m \Longrightarrow Y=0 \Longrightarrow$ deflection نعوض بها فى معادلة ال

at $X = 0\ m \Longrightarrow Y^{\backprime}=0 \Longrightarrow$ Slope angle نعوض بها فى معادلة ال

AT X = 0

$$EIy=0=-\ (0)^3 + \overset{-ve}{\cancel{(0-2)}}^4 / 12 - 2\overset{-ve}{\cancel{(0-4)}}^2 + 15(0)^2$$
$$- \underset{-ve}{\cancel{(0-4)}}^4 / 12 + C_1 0 + C_2 \Longrightarrow \boxed{C_2 = 0}$$

AT X = 0

$$EIy^{\backprime}\ 0=-\ 3\ (0^2) + (0-2)^3 / 3 - 4(0-4) + 30(0)$$
$$- (0-4)^3 / 3 + C_1$$

$\Longrightarrow$ Slope angle معادلة ال

4 2 2

. $\Longrightarrow$ deflection معادلة ال

و بالتعويض عن قيمة X فى أى من المعادلتين يمكننا أن نحصل على قيمة ال deflection أو ال Slope angle عند أى نقطة .

At point (a)

لحساب ال deflection نعوض فى معادلة ال deflection

at Point a $\Longrightarrow X = 2\ m$

$$EIy = -\ (2)^3 + (2-2)^4 / 12 - 2\overset{-ve}{\cancel{(2-4)}}^2 + 15(2)^2$$
$$- \underset{-ve}{\cancel{(2-4)}}^4 / 12$$

$$y_a = \frac{52}{EI} = \frac{52}{20000} = \mathbf{0.0026m = 0.26\ cm}$$

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At point (b)

لحساب ال *deflection* نعوض فى معادلة ال *deflection*

at Point b $\Longrightarrow$ $X = 4\ m$

$$y_b = \frac{177.33}{EI} = \frac{177.33}{20000} = \mathbf{0.0088 m = 0.88\ cm}$$

At point (C)

لحساب ال *deflection* نعوض فى معادلة ال *deflection*

at Point C $\Longrightarrow$ $X = 6\ m$

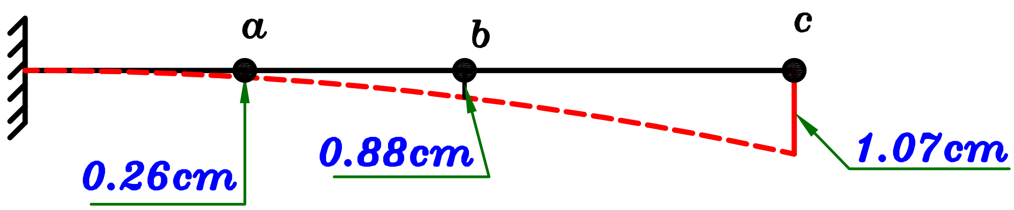
$$y_c = \frac{336}{EI} = \frac{336}{20000} = \mathbf{0.01068 m = 1.068\ cm}$$

At point (C)

لحساب ال*Slope angle* نعوض فى معادلة ال *Slope angle*

at Point C $\Longrightarrow$ $X = 6\ m$

$$y'_c = \frac{82.67}{EI} = \frac{82.67}{20000} = \mathbf{0.00413\ rad.}$$



Elastic Line

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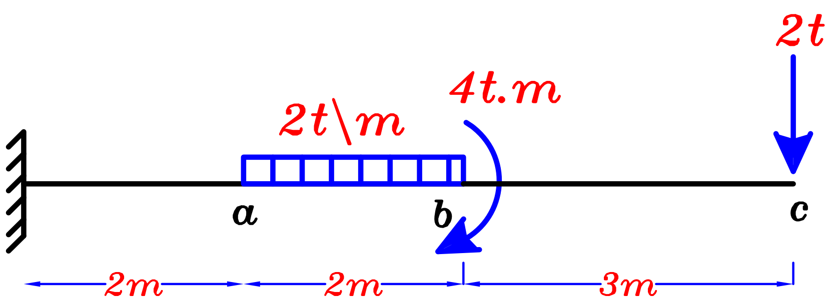
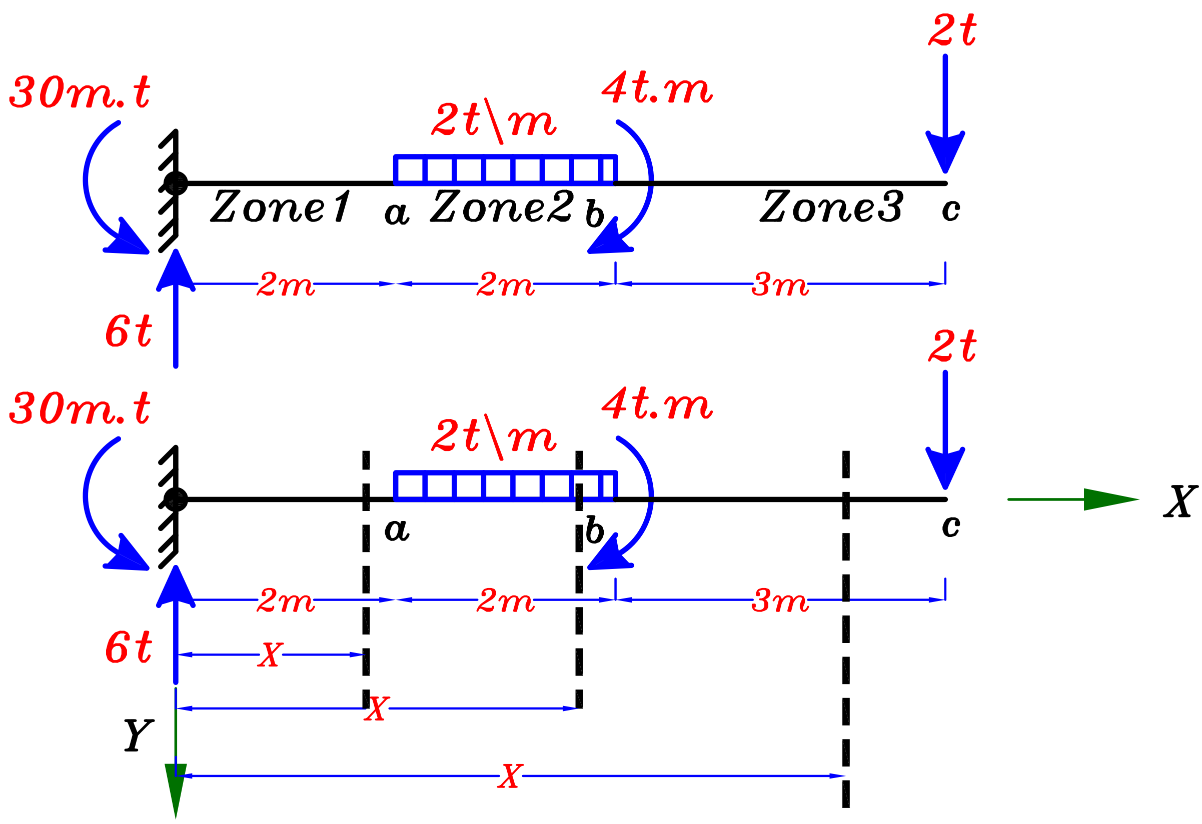
Example:

For the shown beam find the deflection at points (a&b&c) and the slope angle at points (c). applying method of zones ($EI = 20000\ m^2.t.$)





For Zone (1)

$$M(x) = 6(x) - 30(x)^0$$

$$EI\,y'' = -M(x)$$

$$EI\,y'' = 30(x)^0 - 6(x)$$

$$EI\,y' = 30(x) - 3(x)^2 + C_1$$

$$EI\,y = 15(x)^2 - (x)^3 + C_1(x) + C_2$$

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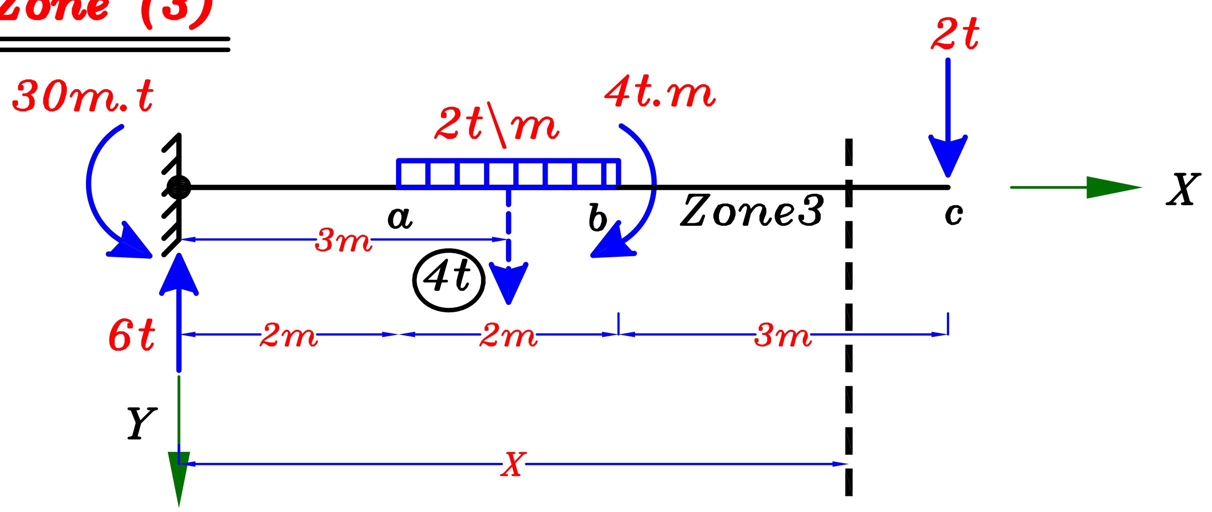
For Zone (2)

$$M(x) = 6(x) - 30(x)^0 - 2(x-2)(x-2)/2$$
$$M(x) = 6(x) - 30(x)^0 - (x-2)^2$$
$$EI\,y'' = -M(x)$$
$$EI\,y'' = 30(x)^0 - 6(x) + (x-2)^2$$
$$EI\,y' = 30(x) - 3(x)^2 + (x-2)^3/3 + C_3$$
$$EI\,y = 15(x)^2 - (x)^3 + (x-2)^4/12 + C_3(x) + C_4$$

For Zone (3)



عند الحساب فى *Zone 3* من الاسهل أن نحول الـ *Distributed load* الى *Concentrated load* و لا نحتاج أن نكمله الى النهاية و ذلك لان هذه المعادلة تعمل فى *Zone 3* فقط أى بعد أن يكون الـ *Distributed load* دخل باكمله.

$$M(x) = 6(x) - 30(x)^0 - 4(x-3) + 4(x-4)^0$$
$$EI\,y'' = -M(x)$$
$$EI\,y'' = 30(x)^0 - 6(x) + 4(x-3) - 4(x-4)^0$$
$$EI\,y' = 30(x) - 3(x)^2 + 2(x-3)^2 - 4(x-4) + C_5$$
$$EI\,y = 15(x)^2 - (x)^3 + 0.667(x-3)^3 - 2(x-4)^2 + C_5(x) + C_6$$

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From intial conditions

at $X = 0$ m $\Longrightarrow$ $Y=0$ $\Longrightarrow$ $\boxed{C_2 = 0}$ $\Longrightarrow$ Zone 1

at $X = 0$ m $\Longrightarrow$ $Y' = 0$ $\Longrightarrow$ $\boxed{C_1 = 0}$ $\Longrightarrow$ Zone 1

---

at $X = 2$ m $\Longrightarrow$ $Y_{a_{Zone1}} = Y_{a_{Zone2}}$

$$EI\, y_{a_{Zone1}} = 15(2)^2 - (2)^3$$

$$y_{a_{Zone1}} = \frac{52}{EI}$$

$$EI\, y_{a_{Zone2}} = 15(2)^2 - (2)^3 + (2-2)^4/12 + C_3(2) + C_4$$

$$y_{a_{Zone2}} = \frac{52 + 2C_3 + C_4}{EI}$$

$$Y_{a_{Zone1}} = Y_{a_{Zone2}} \Longrightarrow 2C_3 + C_4 = 0 \Longrightarrow \text{Eq.1}$$

---

at $X = 2$ m $\Longrightarrow$ $Y'_{a_{Zone1}} = Y'_{a_{Zone2}}$

$$EI\, y'_{a_{Zone1}} = 30(2) - 3(2)^2$$

$$y'_{a_{Zone1}} = \frac{48}{EI}$$

$$EI\, y'_{a_{Zone2}} = 30(2) - 3(2)^2 + (2-2)^3/3 + C_3$$

$$y'_{a_{Zone2}} = \frac{48 + C_3}{EI}$$

$$Y_{a_{Zone1}} = Y_{a_{Zone2}} \Longrightarrow \boxed{C_3 = 0} \Longrightarrow 2C_3 + C_4 = 0$$

$$\Longrightarrow \boxed{C_4 = 0}$$

---

at $X = 4$ m $\Longrightarrow$ $Y_{b_{Zone2}} = Y_{b_{Zone3}}$

$$EI\, y_{b_{Zone2}} = 15(4)^2 - (4)^3 + (4-2)^4/12$$

$$y_{b_{Zone2}} = \frac{177.33}{EI}$$

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$$EI\, y_{b\,Zone3} = 15(4)^2 - (4)^3 + 0.667(4-3)^3 - 2(4-4)^2 + C_5(4) + C_6$$

$$y_{b\,Zone3} = \frac{176.67 + 4C_5 + C_6}{EI}$$

$$Y_{b\,Zone2} = Y_{b\,Zone3} \Longrightarrow 4C_5 + C_6 = 0.667 \Longrightarrow Eq.2$$

---

at $X = 4$ m $\Longrightarrow Y'_{b\,Zone2} = Y'_{b\,Zone3}$

$$EI\, y'_{b\,Zone2} = 30(4) - 3(4)^2 + (4-2)^3/3$$

$$y'_{b\,Zone2} = \frac{74.667}{EI}$$

$$EI\, y'_{b\,Zone3} = 30(4) - 3(4)^2 + 2(4-3)^2 - 4(4-4) + C_5$$

$$y'_{b\,Zone3} = \frac{74 + C_5}{EI}$$

$$Y'_{b\,Zone2} = Y'_{b\,Zone3} \Longrightarrow \boxed{C_5 = 0.667} \Longrightarrow 4C_5 + C_6 = 0.667$$

$$\Longrightarrow \boxed{C_6 = -2}$$

For Zone (1)

$$EI\, y' = 30(x) - 3(x)^2$$

$$EI\, y = 15(x)^2 - (x)^3$$

For Zone (2)

$$EI\, y' = 30(x) - 3(x)^2 + (x-2)^3/3$$

$$EI\, y = 15(x)^2 - (x)^3 + (x-2)^4/12$$

For Zone (3)

$$EI\, y' = 30(x) - 3(x)^2 + 2(x-3)^2 - 4(x-4) + 0.667$$

$$EI\, y = 15(x)^2 - (x)^3 + 0.667(x-3)^3 - 2(x-4)^2 + 0.667(x) - 2$$

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At point (a)

لحساب ال deflection نعوض فى معادلة ال deflection

at Point a $\Longrightarrow X = 2\ m \Longrightarrow$ Zone 1

$$y_a = \frac{52}{EI} = \frac{52}{20000} = \mathbf{0.0026m = 0.26\ cm}$$

At point (b)

لحساب ال deflection نعوض فى معادلة ال deflection

at Point b $\Longrightarrow X = 4\ m \Longrightarrow$ Zone 2

$$y_b = \frac{177.33}{EI} = \frac{177.33}{20000} = \mathbf{0.0088m = 0.88\ cm}$$

At point (C)

لحساب ال deflection نعوض فى معادلة ال deflection

at Point C $\Longrightarrow X = 6\ m$

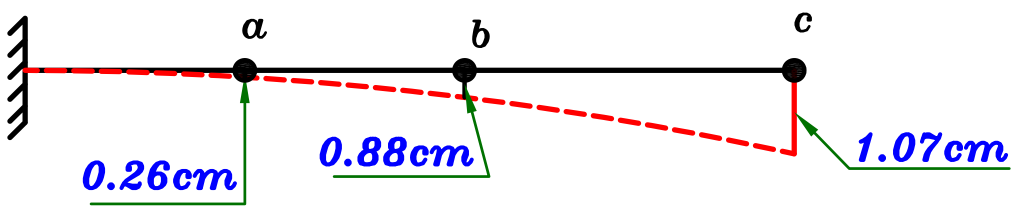
$$y_c = \frac{336}{EI} = \frac{336}{20000} = \mathbf{0.01068m = 1.068\ cm}$$

At point (C)

لحساب الSlope angle نعوض فى معادلة ال Slope angle

at Point C $\Longrightarrow X = 6\ m$

$$y'_c = \frac{82.67}{EI} = \frac{82.67}{20000} = \mathbf{0.00413\ rad.}$$



Elastic Line

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