# Lecture Note
# Applied Mechanics

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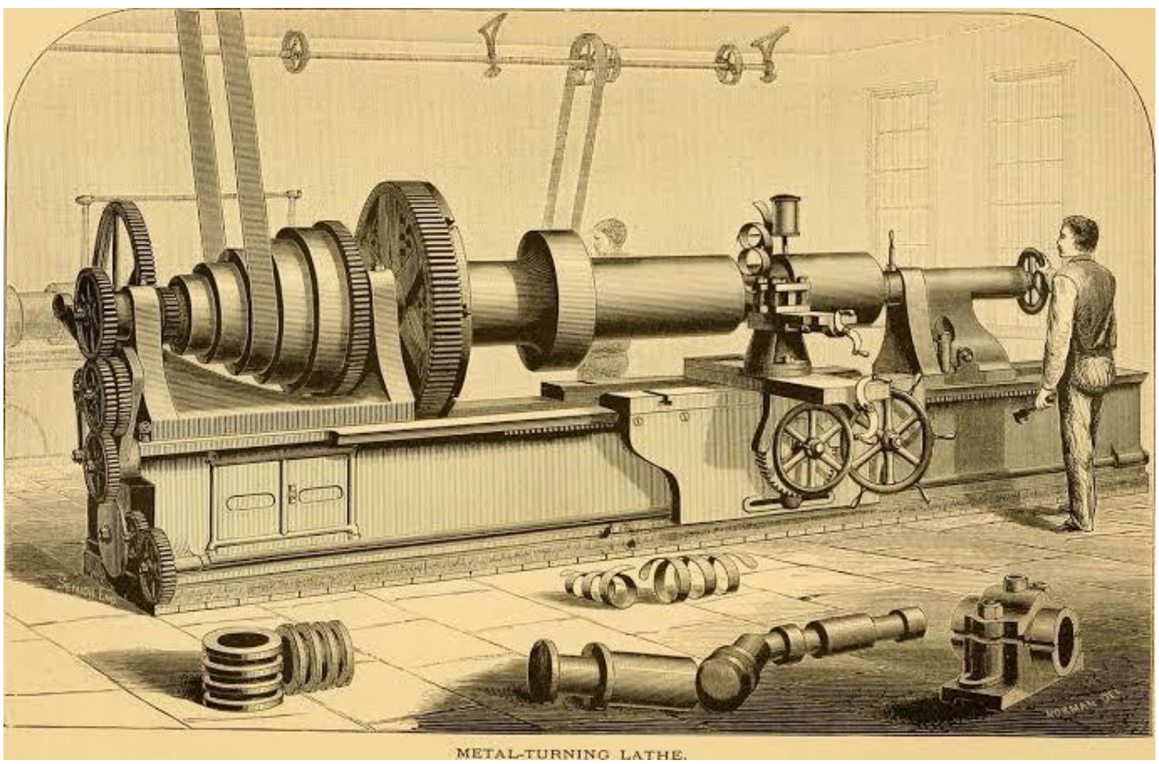
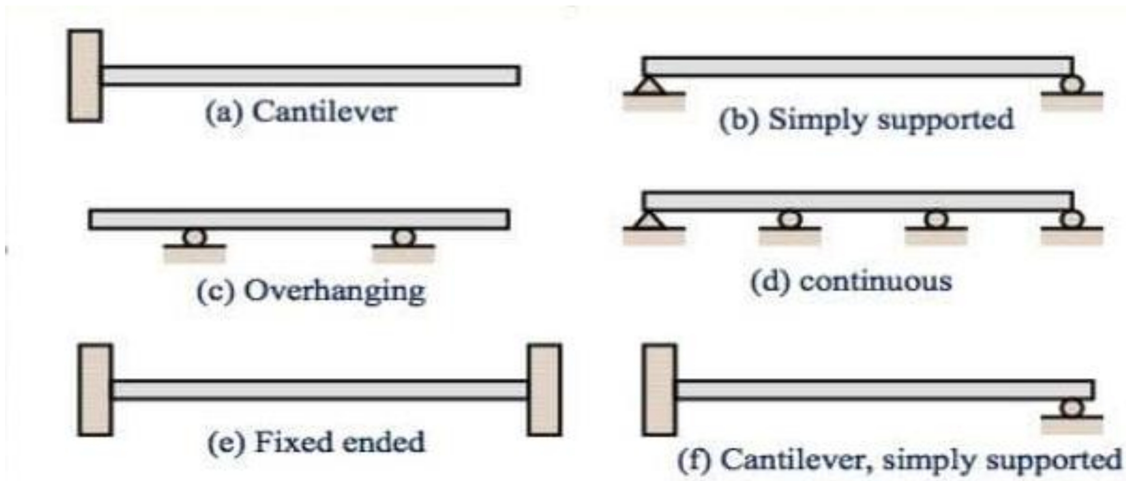
**Mechanical Engineering Department**



# BEAMS

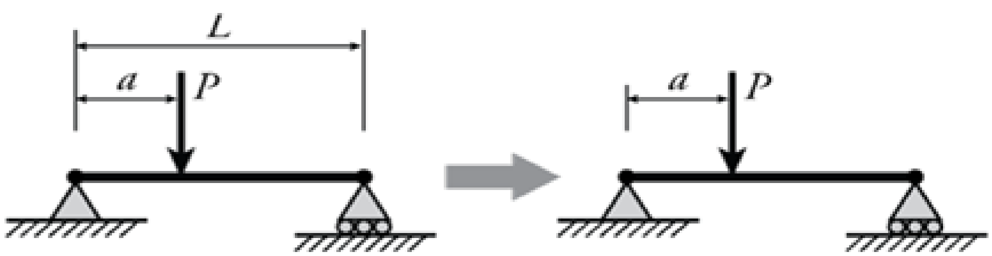
A beam is a structural member or machine component that is designed to support primarily forces acting perpendicular to the axis of the member.

**Beam support classifications:**

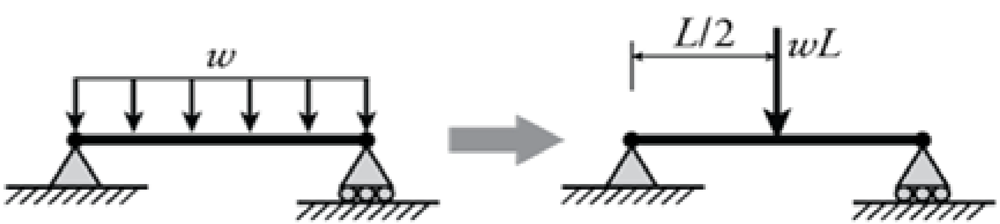



**Types of load :**

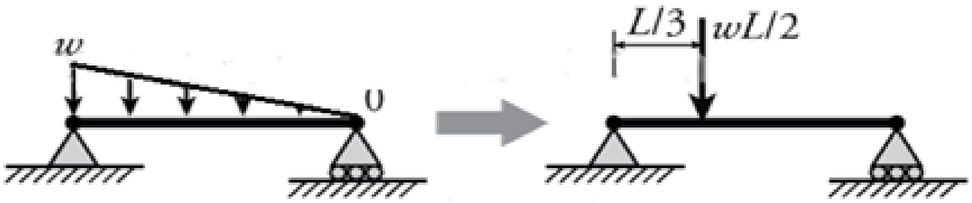
1- **concentrated load**



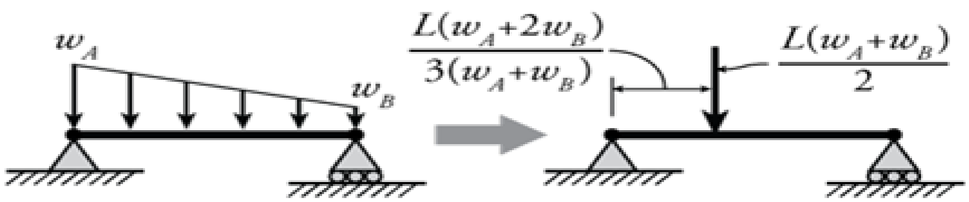

2- **uniformally distributed load:**



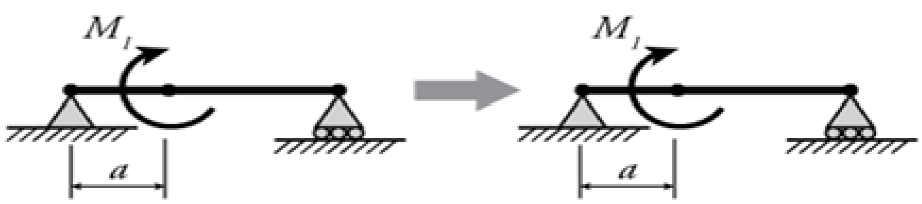
3- **Triangular distributed load:**




4- **Trapizoidally distributed load:**




5- **Point moment:**



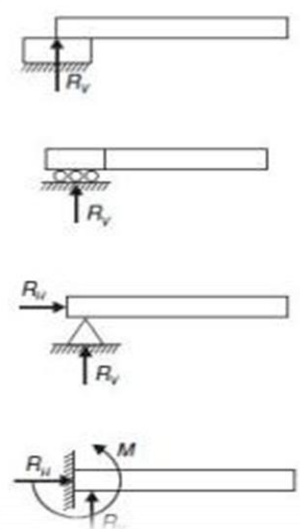

**Types of support:**

1- **simple support**

2- **Roller support**

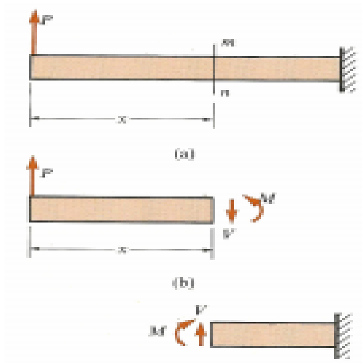
3- **Hinged support**

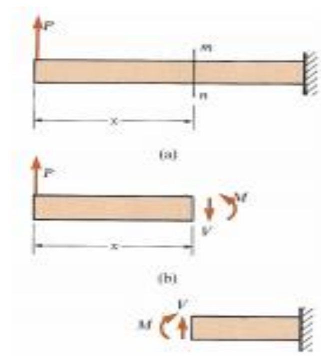
4- **Fixed support**



## Shear Force and Bending Moment:

When a beam is loaded by forces or couples, internal stresses and strains are created. To determine these stresses and strains, we first must find the internal forces and couples that act on cross sections of the beam. Consider the following example:




It is convenient to reduce the resultant to a shear force, V, and a bending moment, M. Because shear forces and bending moments are the resultants of stresses distributed over the cross section, they are known as stress resultants and in statically determinate beams can be calculated from the equations of static equilibrium.

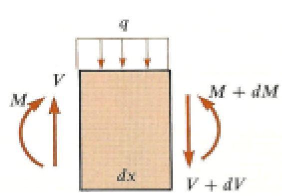
$$\sum FX = 0$$

$$\sum FY = 0$$

$$\sum MA=0$$

## Relationship between Load, Shear Force and Bending Moment:

Consider the following beam segment with a uniformly distributed load with load intensity q. Note that distributed loads are positive when acting downward and negative when acting upward.



Summing forces vertically we find:

$$dV/\,dx = -\,q$$

Thus, the shear force varies with the distance x, and the rate of change (slope) with respect to x is equal to –q. Also, if q = 0, then the shear force is constant.

Rearranging and integrating between two points A and B on the beam we have:

$$dV = -qdx$$

$$\int_A^B dv = -\int_A^B qdx$$

VB – VA = Area of load intensity diagram between A and B

Summing moments and discarding products of differentials because they are negligible compared to other terms, we have:

$$dM/dx = V$$

The above shows that the rate of change of moment with respect to x is equal to the shear force. Also, the above equation applies only in regions where distributed loads act on the beam. At a point where a concentrated load acts, a sudden change in shear force results and the derivative dm/dx is undefined. Rearranging and integrating between two points A and B on the beam we have:

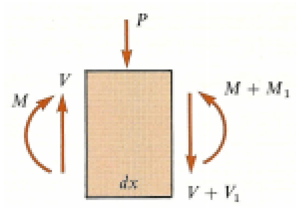
$$dM = Vdx$$

$$\int_A^B dM = \int_A^B Vdx$$

MB – MA = Area of shear force diagram between A and B.

The above can be used even when concentrated loads are acting on the beam between points A and B. However, it is not valid if a couple acts between points A and B.

Now consider the following beam segment with a concentrated load, P. Again, concentrated loads are positive when acting downward and negative when acting upward.




Summing forces vertically we find:

$$V_1 = -P$$

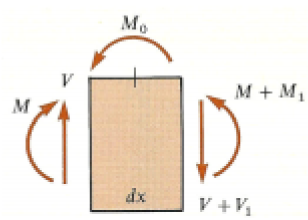
Thus, an abrupt change occurs in the shear force at a point where a concentrated load acts. As one moves from left to right through a point of load application, the shear force decreases by an amount equal to the magnitude of the downward load.

Summing moments we find:

$$M_1 = P(dx/2) + Vdx + V_1dx$$

Since dx is infinitesimally small, we see that $M_1$ is infinitesimally small. Therefore, we conclude that the bending moment does not change as we move through a point of concentrated load application. Recall that dm/dx = V. Since the shear force changes at the point of load application, we conclude that the rate of change dm/dx decreases abruptly by an amount equal to P.

Finally, we consider the following beam segment with a concentrated couple, M0. Counterclockwise couples are considered to be positive and clockwise couples are considered to be negative.



Summing moments and disregarding terms that contain differentials, we have: M1 = -M0

The previous equation shows that there is an abrupt decrease in the bending moment in the beam due to the applied couple, M0, as we move from left to right through the point of load application.

In summary:

**Distributed loads**

Shear force slope (dV/dx) = -q

VB – VA = Area of load intensity diagram between A and B

Moment slope (dM/dx) = V

MB – MA = Area of shear force diagram between A and B

**Concentrated loads:**

Shear force slope (dV/dx) = 0

At load application V1 = -P

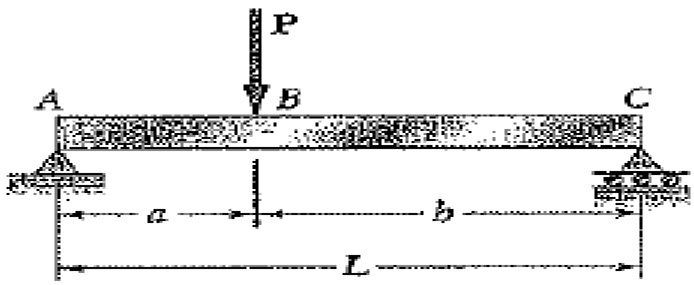
Moment slope (dM/dx) = V (not valid at load application)

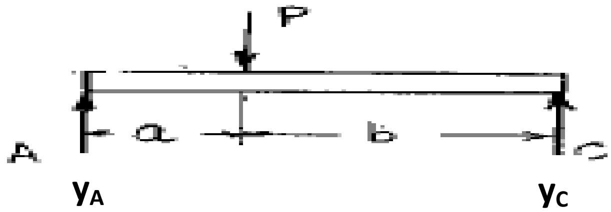
**Concentrated loads:**

Moment slope (dM/dx) decreases by P

MB – MA = Area of shear force diagram between A and B.

**Example1:** Find the reactions at A and C (P=20 KN, a=2m, b=4m)




$\sum FX = 0$

XA=0

$\sum FY = 0$

YA+YC-P=0 $Y_A+Y_C=20KN$

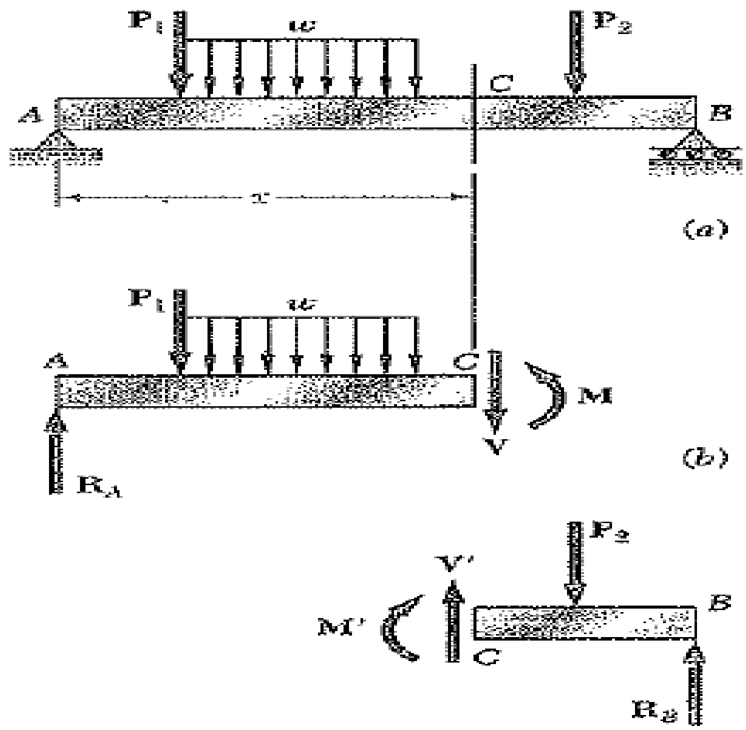
$\sum MA=0$ $20*2-Y_C*6=0$

We get $Y_A$, $Y_C$

## Shear and Bending Moment Diagrams :

As indicated in. as fig, the determination of the maximum absolute Values of the shear and of the bending moment in a beam are greatly Facilitated if *V* arid *M* are plotted against the distance *x* measured from One end of the beam, the knowledge of *Mas* a function of axis essential to the determination of the deflection of a beam·

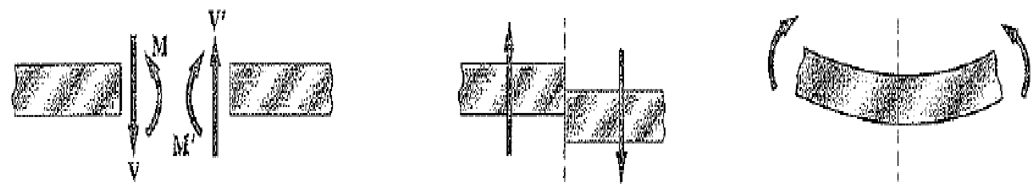
In the examples and sample problems of this sect1on, the shear and Bending-moment diagrams will be obtained by determining the values of Y and *M* at selected points of the beam. These values will be found in the usual way, i.e., by passing a section through the point where they are to be determined Fig (a). And considering the equilibrium of the portion of beam located on either side of the section Fig (b). Since the shear forces **V** and **V'** have opposite senses, recording the shear at point *C* with an up or down arrow would be meaningless, unless we indicated at the same time which of the free bodies *AC* and *CB* we are considering. For this reason, the shear *V* will be recorded with a sign: a *plus sign* if the shearing forces are directed as shown in Fig (b) and a *minus sign* otherwise. A similar convention will apply for the bending moment *M.* It will be considered as positive if the bending couples are directed as shown in that figure, and negative otherwise. Summarizing the sign conventions we have presented, we state:




The shear V and the bending moment Mat a given point of a beam are said to be positive when the internal forces and couples acting on each portion of the beam are directed as shown in Fig (d).

These conventions can be more easily remembered if we note that:

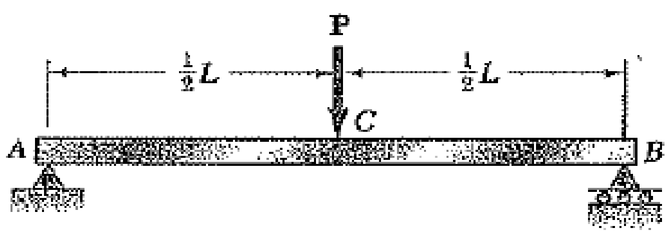
1- The shear at any given point of a beam is positive when the external forces (loads and reactions) acting on the beam tend to shear off the beam at that point as indicated in Fig (e).
2- The bending moment at any given point of a beam is positive when the external forces acting on the beam tend to bend the beam at that point as indicated in Fig (f).




d Internal forces (positive shear and positive bending moment)
e Effect of external forces (positive shear)
f Effect of external forces (positive bending moment)

## Example2:

Draw the shear and bending-moment diagrams for a simply supported beam *AB* of span *L* subjected to a single concentrated load Pat it midpoint *C*.

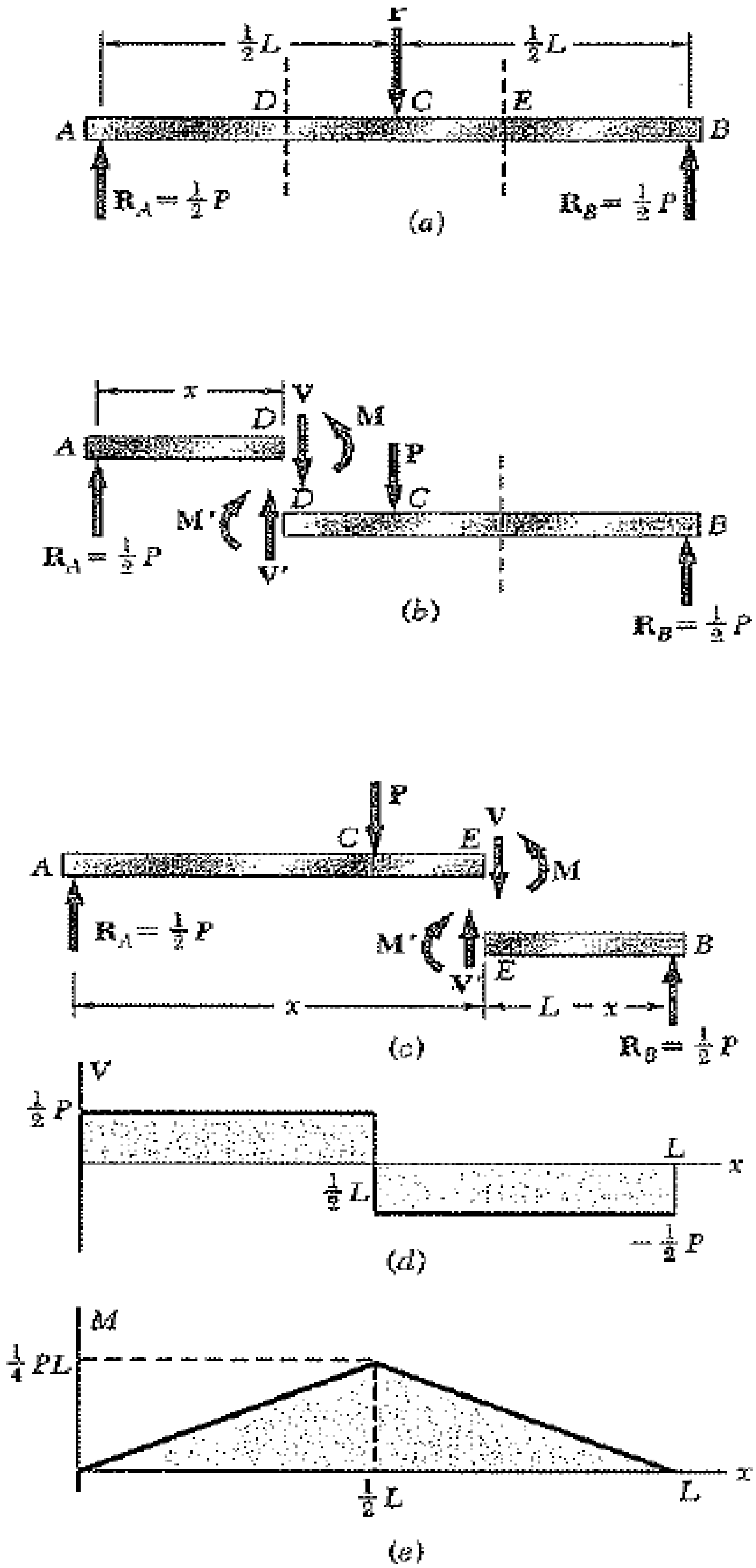



$\sum FX = 0$ XA=0

$\sum FY = 0$ $Y_A + Y_B - P = 0$ $Y_A + Y_C = P$

$\sum MA = 0$ $P*L/2 - Y_B*L = 0$

We get $Y_{A=}$ P/2 $Y_B = P/2$




Draw the shear and bending-moment diagrams for a cantilever

**Example3:**

Draw the shear and bending-moment diagrams for a cantilever beam *AB* of span *L* supporting a uniformly distributed load *w*.

$\sum FY = 0$ $-WX - Y_B = 0$ $Y_B = -WX$

$\sum MA=0$ $WX*(X/2) + M=0$

$Y_B=V_B = -WL$ $MB=-(1/2)*WL^2$

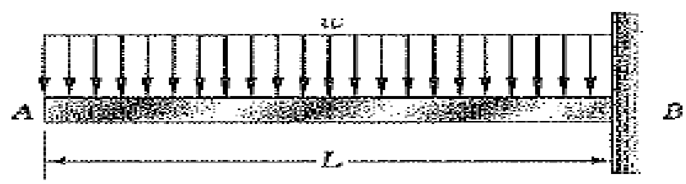
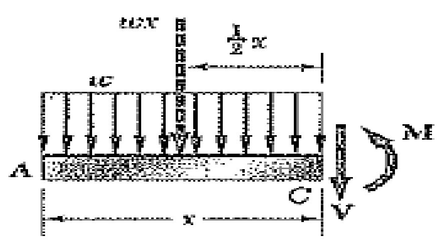
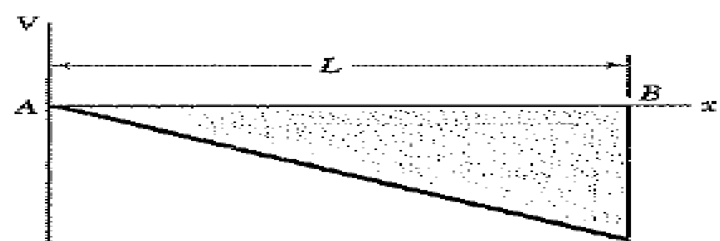


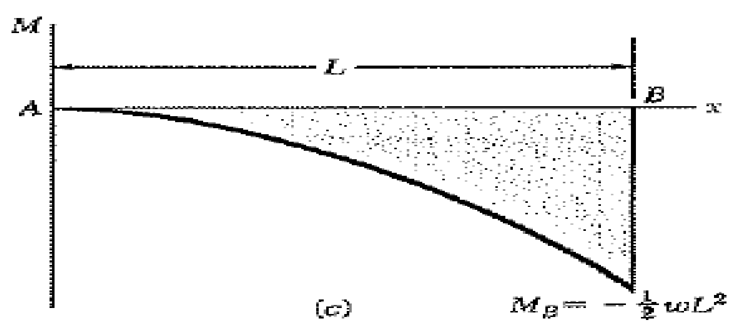

Fig. 5.10




(a)




(b)



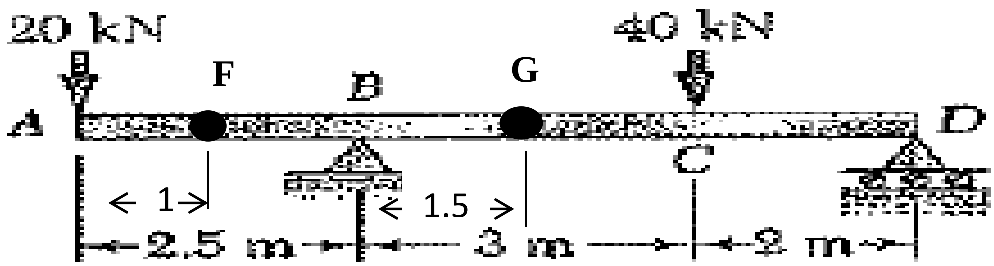

(c)

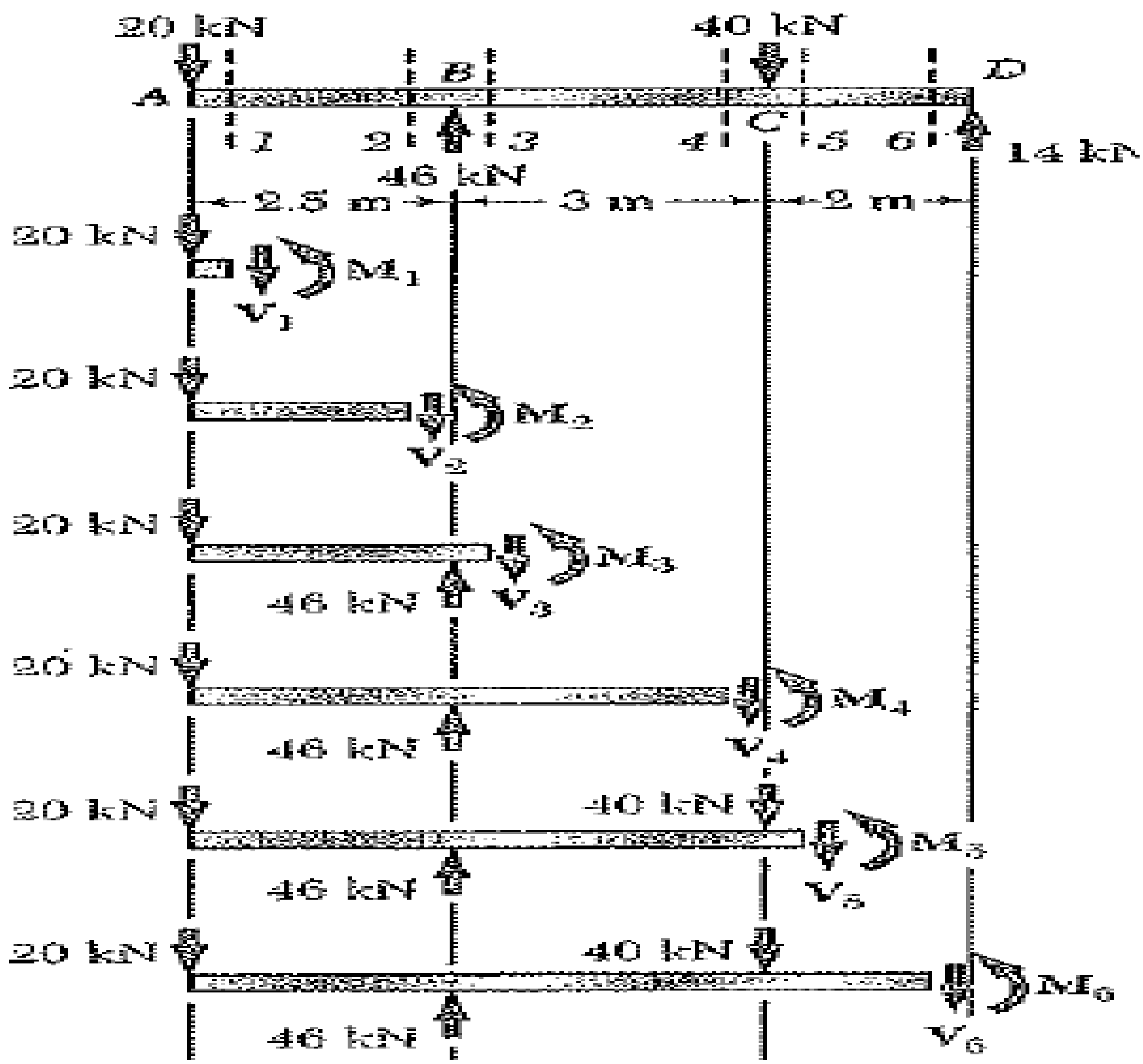
**Example 4:**

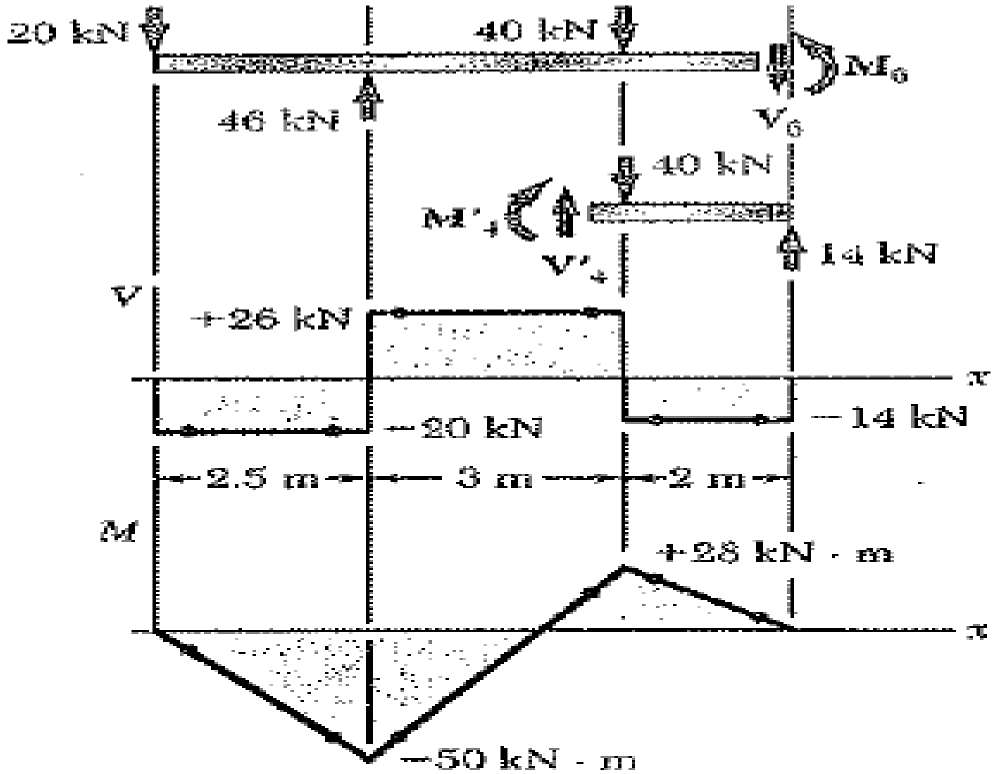
Find the reaction at **B,D**

Draw the shear and bending-moment diagrams

Find the force and moment at point **F, G**




20 kN
40 kN
46 kN
$M_6$
$V_6$
40 kN
$M'_4$
$V'_4$
14 kN
V
+26 kN
x
−20 kN
−14 kN
2.5 m
3 m
2 m
M
+28 kN · m
x
−50 kN · m

**SHEET (1)**

For the beam and loading shown:
Draw the shear force diagram bending Moment diagram as result of( a ,b, L, P or W)

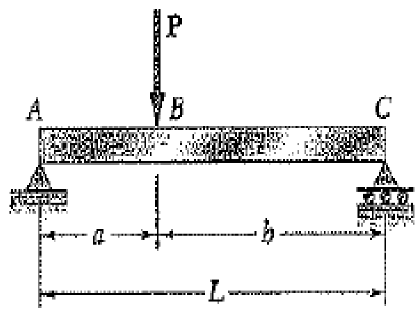


Fig. P5.1

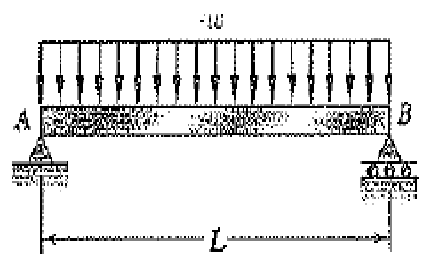


Fig. P5.2

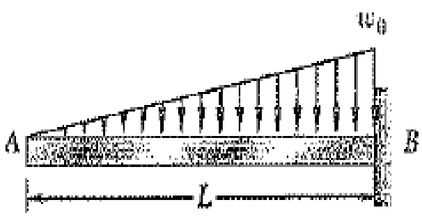


Fig. P5.3

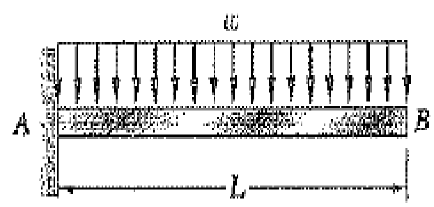
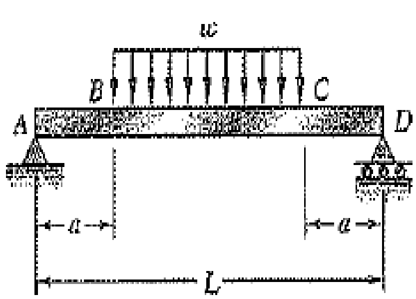
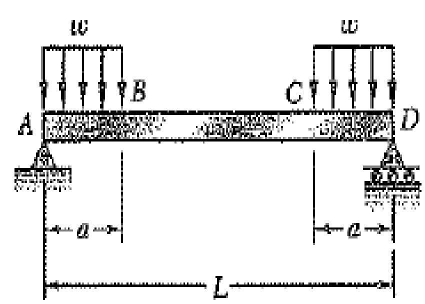


Fig. P5.4



Find the reactions at the supports

Draw the shear and bending Moment diagrams for the beams

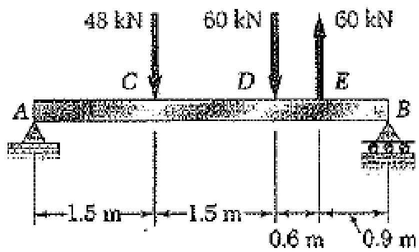



Fig. P5.7

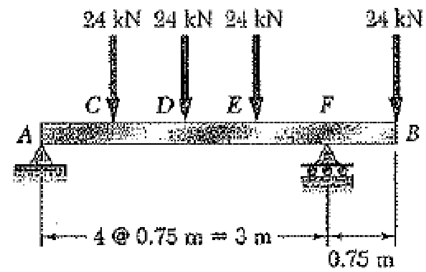



Fig. P5.8